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AUTHOR Poage, Melvin
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ABSTRACT

This booklet contains a series of articles outlining the problem with developmental mathematics training in the United States, with comments from numerous educators and high-ranking business executives. Solutions to improving mathematics training are offered as the result of research from the Continuous Sequence in Basic Mathematics (CSBM) project. The solution is described as a critical thinking approach to teaching mathematics. Changes needed to correct the perceived problems of developmental math include emphasis on reading mathematics, restructure and reduction of curriculum, and more emphasis on critical thinking and problem-solving skills. The booklet offers a 30-unit curriculum, examples of student misconceptions of algorithms, and characteristics of critical thinking. Examples of exercises that utilize critical thinking are provided, including those for algebra, geometry, and problem-solving to reinforce the use of a calculator or computer. Teaching methods for the use of critical thinking are described, including evaluation of students using the process. (AIM)

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ED 399 188

CRITICAL THINKING APPROACH TO MATHEMATICS

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BASED ON THE CSBM RESEARCH

Dr. Melvin Poage

President M. T. E. Ltd.

Materials and Technology for Education

A Mathematical Consultant Company

DENVER, COLORADO

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EDUCATIONAL REFORM FOR THE 21st CENTURY

IF EDUCATION IS TO ACCOMPLISH
A POSITIVE CHANGE IN STUDENT LEARNING,
THEN OUR TEACHING MUST IMPROVE A STUDENT'S ABILITY TO "THINK AND PRODUCE POSITIVE OUTCOMES".

A STUDENT'S ABILITY TO "THINK AND PRODUCE POSITIVE OUTCOMES" IS CRITICAL THINKING.

IN THE PURSUIT OF ACADEMIC EXCELLENCE

WHY CRITICAL THINKING?

Adapted from Academic Excellence

We need Critical Thinking in the classroom because we cannot simply ask or order students to think. Students do not begin class with intellectual standards to think; they are not geared for academics that involve thinking. They do not understand how and why they think as they do, nor how to direct, redirect or assess their thinking that leads to the understanding of Developmental Math or the solution of math problems. They do not naturally think critically or creatively. They are not accustomed to figuring things out or reasoning through the math they study.

On the contrary, students are generally accustomed to passivity and low level thinking performance. Most have never heard of -- let alone learned -- the arts of critical reading, writing, speaking, or listening; they have no conception of what discursive reasoning is. Most have spent the years in mathematics courses merely exercising their rote memories, struggling to store up, at least temporarily, undigested bits and pieces of math information for the next test. Within a few days little, if any, of this information will remain. The mindless ritual of rote memorization has come to stand, in their minds, for math education. Consequently, they are commonly bored, alienated, and poorly motivated. What today's students have learned best are the arts of passive resistance and an aversion to anything academic or scholastic.

Students need to learn how to read mathematics critically, to precisely formulate questions, define contexts and purposes, pursue any needed relevant information, and analyze math concepts, all leading to the positive solution of a problem or situation. These are the qualities that enable good math solvers to figure out and solve problems. The research completed for the Critical Thinking Approach to Mathematics has determined that it is possible not only to introduce students to critical thinking in developmental math courses, but to use critical thinking successfully in more advanced university courses.

THIS BOOKLET IS THE COMPLIMENT of MTE Ltd.

An Educational Math Consultant Company

Dr. Melvin Poage
President of MTE

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Section 1 Comments and Reactions to the Problem

WHY STUDENTS HAVE TROUBLE LEARNING MATH

TEXTS: TOO MUCH STRESS ON ROTE MEMORY, TOO MUCH NONESSENTIAL MATERIAL PRESENTED, THE MATERIAL PRESENTED IS NOT WELL EXPLAINED, STUDENTS CANNOT READ & UNDERSTAND THE TEXT.

CLASS-ROOM: STUDENTS ARE NOT INVOLVED IN THE LEARNING PROCESS - THEY MOSTLY MEMORIZE FACTS, TOO MUCH STRESS ON ACCELERATION. TESTS MAINLY REQUIRE ALGORITHM APPLICATIONS; REASONABLENESS, HYPOTHESIZING, ESTIMATION, AND RECONSTRUCTABILITY ARE NOT EMPHASIZED.

THE PROBLEM

TOO MUCH STRESS ON ROTE MEMORY IN THE TRADITIONAL APPROACH TO TEACHING MATH. WHILE MEMORY DOES PLAY AN IMPORTANT PART IN THE LEARNING PROCESS IT IS NOT POSSIBLE TO MEMORIZE AN ENTIRE SUBJECT (SUCH AS MATHEMATICS). MATERIAL THAT IS MEMORIZED TO PASS A TEST IS SOON FORGOTTEN LEAVING A VOID OF INFORMATION, PARTICULARLY IF THE MEMORIZED MATERIAL IS NOT IN THE STUDENT'S FIELD OF INTEREST.

THE DILEMMA

CURRICULUMS MANDATE THAT TOO MUCH MUST BE PRESENTED TOO FAST; THEREFORE, THE EASIEST WAY TO TEACH MATH IS BY ROTE MEMORY; THE EASIEST WAY TO LEARN MATH IS BY ROTE MEMORY; AND THE BEST TEST RESULTS IN MATH ARE OBTAINED BY ROTE MEMORY TESTS WHICH ARE GIVEN IMMEDIATELY AFTER PRESENTATION OF MATERIAL.

In recent years Japan has had a large percent of their young people, particularly college graduates, join cults. While not all of these cults are violent as the one that gassed a subway station; it is disturbing to many of Japan's leaders that their young people feel the need to join cults that have goals outside the main stream of Japan's society. Some of their leaders participated in a televised seminar to discuss the reason for this practice. These leaders agreed that one of the main causes was their educational systems' reliance on rote memory to pass courses without any critical thinking skills. Their graduates seem to still be seeking answers.

FURTHER DILEMMA

"The problem is that the people in America who should care most about education don't care. Or at least don't care enough. I refer to the educated elite ... (this includes teachers, instructors, and professors) ... . What are we going to do about this? Step 1, it would seem is to stop kidding ourselves. In short, we have to acknowledge as a nation that we are in this together, the American educational system is failing everyone, from the dullest to the brightest. We have to get on with the task of doing something about it. Something drastic."

A. K. "Scotty" Campbell, Vice Chair of American Research Association, Philadelphia Inquirer

MANY PROFESSORS AND INSTRUCTORS ARE NOT WILLING TO CHANGE THEIR METHOD OF INSTRUCTION OR CURRICULUM BECAUSE A CHANGE MIGHT CAUSE SOME INSECURITY AND POSSIBLY MORE WORK.

THE CURRENT SITUATION IN DEVELOPMENTAL MATH

"We have very good success with the top 20% of the students enrolled in college or university mathematical courses. You have every right to be proud of the top 20% in your school and your part in their mathematics education; after all, many other countries send their top students to the U. S. for advanced education because of the success we have with the top 20%.

Unfortunately, only the top 20% of our graduates can compete on an international level in the business place (Mr. Lou Gerstner, CEO of IBM, TV Interview April, 1996).

An Outstanding job with the top 20%, however, implies that we are not necessarily doing an outstanding job with the other 80% of the mathematical students; the developmental math students. It is this population of students that everyone is complaining about. **(In California, 60% of all students entering colleges or universities need remedial education. Governors Conference on Education, March, 1996)**

Corporate America is now spending more on education than all the money spent for public education, Kindergarten to University, and most of it is on educating or reeducating employees. Many businesses are involved in the education of employees:

Some have started their own K - 12 schools, colleges, or universities.

Some have become involved in joint teaching ventures.

Many have contracted consulting companies to conduct college and other courses for employees (this new type of Education Consulting Service is the fastest growing segment of today's economy).

Businesses are spending this large amount of time and money on education because they need better educated employees to improve production and reduce costs in order to keep up and be competive in the new, and constantly changing, international business conditions.

"Not only do graduates need to know and understand basic developmental subjects, but they need to take and pass higher level math, computer, and science courses in order to be competive for the higher paying jobs in the new economy. We (businesses) can train our employees the skills they will need on specific jobs; however, we are not prepared or equipped to teach the basic skills of reading and math needed by every citizen."

Mr. Lou Gerstner, CEO of IBM, Governors Conference on Education, March, 1996

"It is well documented that the need for remediation in mathematics is the most common problem at public colleges and universities (Albers, Anderson, & Loftsgarden, 1987) and that the number of students who enroll in remedial mathematics classes has increased extensively over the past decade (Wepner, 1987) and will continue to increase in the near future."

Writing to Learn Mathematics: Enhancement of Mathematical Understanding,
Gangull, Aparna, AMATYC Review Fall 1994

We in developmental math must not be so afraid of change that we are willing to live with traditional failure; failure caused principally by acceleration (too much, too fast) which, in turn, causes students to memorize instead of understand. We must lead in initiating and making changes in education, and we must not allow educational formats to be dictated by outsiders.

ECONOMICS AND EDUCATION

Those who think that the wealth and well-being of nations of the future is given by the strength of their corporations or the mass of capital available to their wealthy to invest are thinking in terms of categories no longer in keeping with global economic realities. ...

If we have workers who are good critical thinkers -- hence can identify, analyze, and solve problems in collaboration with one another -- capital will inevitably be drawn to us, transforming our work force, as a result, to yet higher levels of skill and insight, which entails yet further capital being drawn to us. On the other hand, if our work force is not characterized by a high level of critical thinking and problem solving, then so-called **"American" corporations will invest their capital in those nations where the high level skills essential to high value production are to be found.** ...

Corporations, becoming increasingly internationalized, will succeed or fail in ways fundamentally independent of the well-being of the home country in which they are theoretically "rooted." Hence, "A foreign-owned firm ... that contracts with Americans to solve or identify complex problems helps Americans far more than an American-owned firm that contracts with foreigners to do the same." ... **"The standard of living of a nation's people increasingly depends on what they contribute to the world economy – on the value of their skills and insights.**

21st Century Capitalism, THE WORK OF NATIONS, Dr. Robert Reich

National education policies are always related to the perceived economic demands of the society. On the one hand, many if not most students view education as a stepping stone to a good job. On the other hand, most business people look to education to produce the kind of workers and work force which will serve the economic needs of the time, as they (business people) perceive them. ...

As educators, either in the formal or informal sense, we have a special interest in understanding global changes in the world and the realities they signal. We can no longer assume that reality will take care of itself. ... **The single most important factor to success is a high level of educational skills in the work force.** ...

HEAD TO HEAD: Economic Battle Among Japan, Europe, and America, Dr. Lester Thurow

Everyone who thinks recognizes the impact of economic forces in our lives. Depression or inflation, few or many jobs, little or much public funds, all that involves money plays a major role in our lives. ...

The fundamental characteristic of all capitalist economies, and of capitalism generally as a world historic force is "kaleidoscopic changefulness," a "torrent of market-driven change." **"If capitalism is anything, it is a social order in constant change – and beyond that, change that seems to have a direction, an underlying principle of motion, a logic."** This logic is the logic of "creative destruction, the displacement of one process or product by another at the hands of giant enterprize." ... With kaleidoscopic change, with the continual social transformations that accompany those changes, come "both wealth and misery, development and damage, **a two-edged sword" that makes for permanent possibility of unpredicted instability.** ...

21st Century Capitalism, Dr. Robert Heilbroner

"The problem of knowledge, freedom, and productivity requires that we, for the first time in history, take true intellectual discipline and the "fitness" of our minds seriously. We must create new conditions in school and society under which intellectual virtues long ignored -- intellectual courage, humility, perseverance, integrity, faith in reason, and fairmindedness -- can develop. **We must learn to be comfortable with, indeed to value, for the first time, rational self-criticism.** We must , for the first time, begin to devote as much time to intellectual habits as we now do to physical ones, and admit, finally, that rationality and openness of mind are not automatic or "natural" states that can be left to themselves to emerge and flourish. To begin to do this, we must reconceptualize the nature of teaching and learning in every context of life. ...

In short, knowledge, freedom, and social progress are deeply intertwined. ... Only through slow, painful change, with much frustration and circling about, only with multiple misunderstandings and confusion, will we work our way, eventually, into rational lives in rational societies."

CRITICAL THINKING: What Every Person Needs to Survive in a Rapidly Changing World, Dr. Richard Paul

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DOES MATHEMATICS INSTRUCTION NEED CHANGE?

Remarks by John Sculley
Chairman and CEO Apple Computer, Inc.
to
President-Elect Bill Clinton's
Economic Conference, Little Rock, Arkansas

December 15, 1992

We are at a major turning point in the economy. This is a change as significant as the transformation of our 19th Century agricultural economy into the 20th Century industrial economy which took place a century ago.

The hallmark of this 20th Century industrial economy has been mass production and mass consumption. The mass production / mass consumption model of business was pioneered in this country by early 20th Century leaders like Henry Ford and the industrial engineer Fredrick Taylor. This model assumed that all important decisions were centralized around a small managerial elite. **"Thinking" was purposefully separated from "doing".** Work was broken down into repetitive tasks, and management consciously eliminated as much decision-making as possible for the workers. Quality was defined around rigid standardization. There was little flexibility in the work process.

Perhaps the biggest change in this decade will be the reorganization of work; that is, re-engineering the ways in which work gets done productively. But here is a major problem: Most Americans don't know what the New Economy is, yet it will dramatically impact their lives in the years ahead. Today we measure economic health on internal comparisons like the GDP (Gross Domestic Product) growth and unemployment rates. In the New Economy, the key indicator is standard of living, as measured externally by comparative productivity versus that of other industrialized trading regions. **Our high standard of living already requires higher productivity than the rest of the world.** Fortunately, we are still ahead. But we are losing ground. Real income is down. Only the best-educated, higher-income families are holding their own.

In the New Economy, that old industrial model is being replaced by a rapid move towards customization of goods and services and the decentralization of work. Today, new products can be developed in a fraction of time it took in the old industrial economy; services and products are being custom built to order; quality is dramatically improving; and costs are being driven down through the use of new technologies such as computer systems, robotics, and measurement systems.

In the New Economy, workers are on the front-line interacting with customers, and workers on the factory floor are empowered to make decisions. This is the only way customized goods and services can be created quickly, with the highest quality, at the lowest cost, with maximum flexibility.

In the old economy, America had a real advantage because we were rich with natural resources and our large domestic market formed the basis for economies of scale. **In the New Economy, strategic resources no longer just come out of the ground (such as oil, coal, iron, and wheat); the strategic resources are ideas and information that come out of our minds.**

DOES MATHEMATICS INSTRUCTION NEED CHANGE?
Remarks by John Sculley (Con't)

The result is, as a nation, we have gone from being resource-rich in the old economy to resource-poor in the New Economy almost overnight! Our public educational system has not successfully made the shift from **teaching the memorization of facts** to achieving the **learning of critical thinking skills**. We are still trapped in a public educational system which is preparing our youth for jobs that no longer exist.

The New Economy is global. We are no longer alone at the top. In fact, the United States is underprepared to compete with many other major industrialized trading regions in the world. **Students in other industrialized countries are learning math, and critical judgment skills that are more relevant to the New Economy.**

Other industrialized countries have an alternative path for the non-college-bound, including vocational study and a school-to-work transition which is tightly linked with apprenticeships and worker training in industry. We have few alternatives for the non-college-bound students so they can participate productively in the high skilled work of the New Economy.

The greatest certainty about the New Economy is the pace of change. Young people in school today can reasonably expect to have four to five careers. Skill needs will constantly change, as well. **Education, therefore, must become a lifelong pursuit -- not just an institutional experience early in one's life.** Education, training, and retraining must become as much an ongoing experience in our lives as exercise and vacations.

Most Americans see our largest corporations going through massive restructurings, layoffs, and down-sizing. People know something has changed and they are scared because because they don't fully understand it and they see people they know losing their jobs. They also see their neighbors buying high-quality, lower-priced products from abroad, and they ask why can't we build these same products or better ones here at home?

The answer is, we can. But only if we have a public education system which will turn out a world-class product. We need an education system which will educate all our students not just the top 15-20 percent.

A highly-skilled work force must begin with a world-class educational system. Eventually, the New Economy will touch every industry in our nation. There will be no place to hide. In the New Economy, low-skilled manual work will be paid less. The United States cannot afford to have the high-skilled work being done somewhere else in the world and, as a result, end up with the low-wage work.

This is not an issue about protectionism. It is an issue about an educational system aligned with the New Economy and a broad educational opportunity for everyone.

The reorganization of work into decentralized, higher-skilled jobs is the systemic key to a vital American economy in the future. We are talking about the standard of living that we, and our children, and their children will have well into the 21st Century.

It's America's Choice: High skills or low wages.

MATH AND SCIENCE SKILLS
ESSENTIAL TO YOUNG ENGINEERS

Remarks by Kenneth T. Deer
Chairman and CEO Chevron Corporation
to
National Engineers Week Conference

South San Francisco, February 23, 1996

I remember when the slide rule was an essential classroom tool. Today, the classroom wouldn't be complete without a computer. I remember watching the Sputnik launch, awed like the rest of the world by the first ever artificial satellite launching. Now endeavors in outer space are almost common. In just a short time, I have felt the effects of numerous, substantial engineering advancements. **Advancements that have changed the way we live - even how long we live.** From cellular phones and Internet searches to organ transplants and artificial limbs, engineers contribute to nearly every major technological and medical advancement.

In 1996, it's difficult for any of us to imagine life without everyday conveniences like telephones, refrigeration and motorized vehicles. So, as another National Engineers Week passes, it's important for all of us to reflect on the significance of this field and look toward the future of engineering. Technology will continue to be the driving force in our global marketplace. **And America's future competitiveness depends on the quality of our students.** We at Chevron are constantly searching for well-educated engineers, and like many other U.S. corporations we can only benefit from cultivating a strong engineering workforce here at home.

We need our children to excel in math and science so that they may continue to make advances in medicine, academia and industry. The cures for AIDS and cancer, disposal of nuclear waste and energy conservation are all issues that will be taken on by our children. **Yet, according to recent national exams, three-quarters of U.S. graduates lack basic math skills.** And the National Center for Research in Mathematical Sciences Education says that American high school students routinely rank lower than European and Asian students in comprehensive science exams.

As a corporation, Chevron has developed several programs to help combat this decline. We realize how important it is for parents, teachers, corporations to team-up to help motivate our nation's children to pursue math and science studies. Engineering is essential, interesting work. With disciplines that range from mechanical, civil and chemical to electrical, environmental, and biotechnical, there are numerous options available to be an engineer. And from a financial standpoint, you couldn't pick a better career. Engineers earn an average of $60,000 - $90,000 per year. The average starting salary for graduates with a bachelors degree in engineering is higher, and unemployment lower, than almost any other field.

Most people become engineers because they enjoy math and science. And most students who enjoy math and science do so because they were encouraged in their efforts. My own pursuit of a mechanical engineering degree was largely the result of my father's encouragement. When children ask questions like, "Why is the sky blue?" and "How does the television work?", **we must foster their curiosity, teaching them how to find their own answers, make their own discoveries.** These discoveries will help us in the decades to come.

The U.S. has always been a strong force in the global marketplace, and we will continue to be as long as we focus on the future. There is nothing more essential to the future of engineering than the education of our youth. **We must work together to ensure sound math and science educations of our children.**

ADULT REMEDIAL COURSES

Remedial courses in public schools are NOT new; however, adult remedial courses taught in colleges and universities are a new educational area. Since the college and university environment is so completely different from that of public schools, it is not surprising that adult remedial courses and the texts for those courses should differ greatly from those in the schools. Yet many of the existing texts for adult remedial courses are no more than a rehash of elementary school texts. We are now in a period of mathematics education where most of the students entering college or university do not have sufficient backgrounds in mathematics to pursue degrees that will benefit their futures. We are faced with a long-term situation and we must meet this challenge.

"It is well documented that the need for remediation in mathematics is the most common problem at public colleges (Albers, Anderson, & Loftsgaarden, 1987) and that the number of college students who enroll in remedial mathematics classes has increased extensively over the past decade (Wepner, 1987)."

Gangull, Aparna, *Writing to Learn Mathematics: Enhancement of Mathematical Understanding*, The AMATYC Review, Fall 1994

A KEY QUESTION IS , -- AT THE COLLEGE AND UNIVERSITY LEVEL, DO WE NEED TO TEACH DEVELOPMENTAL MATHEMATICS ? SURVEYS INDICATE THAT GRADUATES COMING OUT OF TRADITIONAL MATHEMATICS COURSES CANNOT APPLY THEIR MATHEMATICS KNOWLEDGE TO NEW SITUATIONS THEY ENCOUNTER IN BUSINESS OR INDUSTRY.

"...nationally, only about 15% of the 600,000 students who take precalculus or college algebra courses each year ever go on to *start* calculus. Relatively few of these actually complete the calculus sequence successfully."

Sheldon P. Gordon, *Implementing Change in the Mathematics Curriculum*, The AMATYC Review, Fall 1994

I am often asked by my friends, "Why am I trying to make every student a mathematician?".

As a Math Educator you know that there is little likelihood that a student who needs to take developmental math in college or university will become a mathematician. However, for nearly ten years the universities in the United States have not produced enough mathematics PhD's to fulfill all the needs of industry, research, government, and education (although "down-sizing" has caused a short term reprieve). Recently even some outstanding universities have been forced to consider eliminating or reducing their graduate math programs for lack of eligible student candidates or professors to teach courses. Maybe it's time that we should to try to expand that pool, what a coup - - a student going from Developmental to PhD in math. As our students say, "It couldn't hurt."

In the years between the mid 60's to the mid 70's, America's businesses learned that they must change the way they did business in order to stay competitive in the international marketplace. America's businesses did change and met the challenge; educational institutions have not learned they must change the way they educate students.

James M. Poage, CFO Comtech, Spring 1996

TEACHING and the EMOTIONAL LIVES of STUDENTS

Students should not celebrate their graduations -- as so many do -- by expostulating,

> "Thank God it's over! Thank God I'll never have to take another class or read any of those books or write any more papers or listen to any more lectures! Thank God!"

When this happens we know we, and our students, have failed. The very idea of education implies a special reshaping. When it occurs, students leave school with a mind committed to learning, a mind experienced in both the discipline and joy of developing itself. As educators, we want students not only to acquire some knowledge, but also to genuinely value the getting of it. We want students not only to experience a variety of subjects, but also to change their thinking as a result.

CRITICAL THINKING JOURNAL FALL 1995

RATIONALE FOR CHANGE

According to most resources, jobs, in particular good paying jobs, for the nonmathematical population in the coming highly technical age, will demand a better understanding of basic mathematics, the ability to read mathematics in reports, and the ability to make judgments about solutions to problems. If we are to prepare "nonmathematical" students for this new age, we must improve their preparation in math. Lest we forget, students have paid tuition for a chance to become more educated; we must not let them down because of our lack of effort or unwillingness to try new and different approaches to increase student success.

With few exceptions, each day brings at least one news article or TV release on the disturbing state of American Education, particularly math. There are so many articles and other sources to choose from that it is difficult to know where to BEGIN. Can all of these sources, many of whom are educators, be wrong? Can any educator really believe a problem in college education does not exist? Many nonmathematical students need remediation; **the prognosis for the next few years is that the need of remediation will be even greater than it is now.** The situation is particularly serious if our country attempts not only to keep pace with other countries, but to become the leader in technology. To accomplish this goal the percentage of students who should be taking college or university level math courses must increase. We must increase the number of developmental math students who understand math concepts and the mathematical thought processes (critical thinking) in order to add to the possibilities for increasing this pool.

WE ARE IN A RACE BETWEEN EDUCATION AND CATASTROPHE.

WE MUST EDUCATE STUDENTS TO UNDERSTAND AND APPRECIATE THEIR ENVIRONMENT, BOTH GLOBAL AND UNIVERSAL.

DO WE HAVE ENOUGH TIME?

Buckmister Fuller 1933

THE MORE I AM EXPOSED TO INSANITY,

THE MORE IT APPEARS TO BE NORMAL.

Section 2 CSBM Research Data

Few instructors enjoy teaching Developmental Mathematics courses. It is hard work with little reward. The mathematics is not usually challenging. Our society has made the teaching of these courses a thing to be ashamed of -- or at least something that one doesn't brag about. As a result; most research in developmental math has been limited to improving the status quo with little if any success. In contrast, research involving the top 20% has addressed course organization as well as teaching methods.

SUPPOSE YOU COULD DO A RESEARCH PROJECT IN DEVELOPMENTAL MATH

THAT YOU HAD FUNDING, A VERY CAPABLE STAFF, COMPUTER FACILITIES, AND TIME IS NOT A FACTOR. KNOWING THE SITUATION AS IT EXISTS TODAY:

WOULD YOU START WITH THE IDEA OF IMPROVING THE STATUS QUO?

WHAT AREAS DO YOU THINK NEED TO BE INVESTIGATED?

IF CHANGES WERE INDICATED, WHAT CHANGES MIGHT YOU RECOMMEND?

IN WHAT AREAS COULD YOU EFFECT THE GREATEST CHANGE?

There is a high probability that few of us, if any, would agree on the changes that should be made and how these changes should be implemented -- and that is not bad. Improvement results when different ideas are investigated and tried out.

I was very fortunate to be given an opportunity to be involved in a project that researched Developmental Math. The project was called the Continuous Sequence in Basic Mathematics -- in short (CSBM). The results of this research indicated there is a solution to the developmental mathematics problem; I do not feel that this solution is the only possible one, perhaps not even the best solution, but it is a solution and it works. The CSBM research concentrated on the nonmathematical college student population (not the top 20%, but the bottom 80%).

THE CSBM RESEARCH PROJECT

In 1972 Dr. Wegener and I were asked by our university, Central Michigan University, to develop a new course of studies for the students who were unable to pass the existing mathematics course required for students who needed math, but were not prepared for calculus. The percent of students who were able to pass this course on the first attempt had fallen to about 30% and the professors who were in contact with these students felt that this percent probably would fall even further. Students taking this course were mostly high school graduates, an increasing number of returning older students who had lost their jobs in the auto industry, and some transfer students. Students who needed math had the option of taking calculus or the required mathmatics course. The required mathematics course (a traditional beginning math course) started with a review of the real number system, included topics from both Beginning and Intermediate Algebra, and concluded with topics from College Algebra. The course was a 5 hour course that met 5 days a week with no outside lab available.

After several years of attempting various "band-aid" cures Dr. Wegener and I concluded that a major overhaul of the course, particularly the curriculum and the instructional methods, was needed. We became the directors of the Continuous Sequence in Basic Mathematics (CSBM) research project. The CSBM project lasted from 1974 to 1981, at which time other professors were appointed as directors. Both Dr. Wegener and I left Central Michigan University, I continued the CSBM research at the University of Denver until I retired and I have continued my research in this domain to the present.

As directors of CSBM Dr. Wegener and I developed the curriculum, wrote the course materials, and managed a nontraditional research project involving 200-3000 developmental students, 15 faculty, 50 or more student assistants, and 2 - 4 computer programmers. The Sequence consisted of 30 Units that spanned mathematics from Arithmetic through Precalculus. The main objective of CSBM was to strengthen the mathematics background of nonmathematical students so that they could enroll in one of the calculus courses at the university. CSBM had a success rate of between 80 - 90% of the students who enrolled completed the sequence and 70% of these students, by their own choice, did enroll in one of the calculus courses. Most completed and passed the calculus course. The success of CSBM was recognized by the MAA, NCTM, ERIC, New York Times, and other publications.

The CRITICAL THINKING APPROACH TO TEACHING MATHEMATICS PROGRAM and the accompanying TEXTBOOKS are an outgrowth of the CSBM research project.

RESEARCH OF THE CRITICAL THINKING APPROACH TO TEACHING MATH

THE CHALLENGE

The CHALLENGE for the CSBM researchers was to provide better tools and a better method for teaching the nonmathematical developmental college students; to respond they created the Critical Thinking Approach To Teaching Mathematics, and developed an instructional program to ensure a higher rate of success.

THE RESEARCH RESULTS INDICATED CHANGES

The CSBM research results indicated that certain changes were needed to correct and possibly eliminate the problem in Developmental Mathematics courses:

More emphasis on reading mathematics

Nonmathematical students cannot reconstruct math concepts as needed after completing a math course because of their inability to read math. *Exercises for Vocabulary* were designed and inserted in the Critical Thinking Approach to Mathematics materials to help students learn terminology, to read mathematics, to be able to use reference material, and to reconstruct concepts after college. Materials were written at an appropriate level with understandable explanations.

Restructure and reduction of curriculum

We often try to "jam" too much content in too short a time period. Traditional review courses present an impossible number of concepts to be learned in one term, forcing students to memorize rather than understand concepts; this, in turn, forces much of the following term to be spent in review. A Thirty Unit curriculum spanning arithmetic to precalculus in five terms was developed. This allowed enough time for students to master the topics they needed.

More emphasis on critical thinking and problem solving skills

Nonmathematical students cannot use critical thinking to improve their reasoning. *Exercises for Understanding* containing discussion, explanation, and discovery exercises were designed and inserted to improve critical thinking skills. If one is able to think critically, then problem solving is not a mystique. *Problems for Problem Solving*, which can be used as problem solving tools were designed and inserted in the Critical Thinking Approach to Mathematics materials.

"SUCCESS BREEDS SUCCESS"

PHILOSOPHY OF CRITICAL THINKING APPROACH TO TEACHING MATH

A "NONMATHEMATICAL COLLEGE CAPABLE STUDENT" CAN LEARN AND UNDERSTAND MATHEMATICS, THEN APPLY THIS KNOWLEDGE TO SOLVE PROBLEMS.

Rote memory is not enough. Mathematical understanding, limited to addition of whole numbers, is not enough. Nonmathematical college capable students can understand and learn basic mathematics: Arithmetic, Geometry, Algebra, Pre-Calculus, and some mathematics at the university level. Nonmathematical college capable students can apply their knowledge of mathematical methods and reasoning to solve problems in the real world.

We have met the enemy and they is us! **Pogo**

DATA OF THE CSBM RESEARCH PROJECT

Like most developmental researchers, our first thought was that we simply needed to improve what we were doing -- improve the status quo. Therefore, we began our study, from Winter 73 to Winter 75, by making few organizational changes, but trying to improve our teaching. We kept data on the variables we identified that were related to the learning of developmental math and used the previous course history as a control (shown in the box below). At the end of the Winter 75 semester, the statistics based on this data indicated no growth in understanding, little improvement in tests results, and little positive change in the attitude toward mathematics; shown below.

BEFORE CSBM 1973 -- 1975

	W73	F73	W74	F74	W75
Students	64	84	67	148	132
Passed	21	30	25	68	54
Percent	33%	36%	38%	46%	41%

It became very apparent that a complete overhaul of the course was necessary and based on the research of others we felt that we could make some significant changes in the way developmental math was taught; the CSBM project was started and the first draft was completed in the Summer of 75. From F75 to W 77 the CSBM program was continuously revised and refined. With a completely different organization and approach to teaching developmental math, the results were dramatically improved, shown in the following table.

DEVELOPING CSBM 1975 -- 1977

	F75	W76	F76	W77
Students	466	428	478	452
Passed	359	334	392	384
Percent	77%	78%	82%	85%

In the Winter 77, the Administration of the University recommended that all students entering the university with insufficient math background complete the CSBM sequence. The Math Department voted to implement this recommendation; the project was not prepared for this large influx of students and had to readjust to regain success.

CSBM 1977 -- 1980

	F77	W78	F78	W79	F80
Students	1175	987	1142	938	2331
Passed	905	799	959	807	2029
Percent	77%	81%	84%	86%	87%

Data for the three tables above was collected and compiled by the CSBM research project

I believe othe best indicator of the success of the Critical Thinking Approach was not done by the CSBM research team, but a study done by the Mathematics Association of America. During the late 1970's and early 1980's it was the practice of the MAA to send evaluation teams to investigate and evaluate the strengths of Mathematics Departments of Universities who requested this service. The MAA team that came to our university was unaware of the CSBM research project; and after a study of the research, decided to do a follow-up study to compare the grade point average of the CSBM students in the Business Calculus to that of all students taking that course. The other students, the non - CSBM students, were those that indicated on their placement tests that they had a sufficient math background and did not need the CSBM review.

COMPARISON OF ALL STUDENTS TO CSBM STUDENTS IN BUSINESS CALCULUS

	76 - 77	77 - 78	78 - 79
All Students	860	671	1129
Mean GPA	2.36	2.52	2.60
CSBM Students	44	139	146
Mean GPA	3.33	2.88	2.95

Data collected and compiled by faculty committee under the direction of the MAA.

Conclusion of MAA study of CSBM

It seems to be considerably more useful and beneficial in succeeding courses for students to really understand 70%, than to be exposed to all of the math in a course and have little or no understanding.

We operate under the illusion that coverage is more important than depth - that quantity is more important than quality.

Conclusions from the CSBM Research

There seems to be two main factors that could be corrected in our teaching of developmental math courses that would go a long way toward eliminating the current problem. These two factors are **memorization** and **acceleration.**

Memorization: Students are trying to memorize all necessary facts that are needed to pass the course. Unfortunately, this is not possible for most students and such a course of action has no long term benefits, hence, causing the basic developmental problem in math.

Acceleration: We are attempting to teach too much too fast in developmental math courses. This, in turn, causes students to rely on memorization as their best hope to pass the course. We don't give them the proper amount of time to understand concepts which reduces the need for memory.

TEXT MATERIALS

"... textbooks in this country typically pay scant attention to big ideas, offer no analysis and pose no challenging questions. Instead, they provide a great array of information or 'factlets', while they ask questions requiring only that students be able to recite back the same empty list. ... our fifth finding from research compounds all the others and makes it harder to change practice: teachers are highly likely to teach in the way they themselves were taught. ... by the time we complete our undergraduate education, we have observed teachers for up to 3,060 days. ... We are caught in a vicious circle of mediocre practice modeled after mediocre practice, of trivialized knowledge begetting more trivialized knowledge. Unless we find a way out of this circle, we will continue recreating generations of teachers who recreate generations of students who are not prepared for the technological society we are becoming."

Mary Kennedy, *Policy Issues in Teaching Education,* Phi Delta Kappan, May, 1991.

MOTIVATION AND TEXTBOOK FORMAT

Motivation is the stimulus for learning, and real student motivation is created by a student achieving mathematical success, not from pretty texts. There have been a number of studies about classroom materials and text formats, not one of these studies corroborate the commonly held belief that color, large type, extra space, or cartoons improve student motivation or learning. On the contrary, sufficient evidence indicates that many students are confused or make wrong interpretations because of this Barnum and Bailey type of format; that the use of these extras often act as distracters instead of reenforcers for learning. More importantly -- we state that we are preparing students for successful future employment. How does a circus-like format in classroom materials and texts prepare students to read and understand manuals, reference material, and other company documents produced at minimum cost materials they must read, understand and follow if they are to continue their employment?

The CRITICAL THINKING APPROACH TO TEACHING MATHEMATICS texts were developed for the CSBM math program, these texts were particularly designed to increase student motivation and learning resulting from student success in understanding mathematical concepts. Until now, texts were written for a mathematically interested student who could learn easily in the typical lecture style. The CRITICAL THINKING APPROACH to TEACHING MATHEMATICS texts have been written especially for the nonmathematically inclined student; students who are passive in a lecture approach and whose critical thinking skills are weak. This series of texts is designed to use a shortened lecture of thirty minutes or less and spend the remainder of the class in discussion groups that involve critical thinking skills. These texts do not have a circus format; they do not contain color, large type, extra space, or cartoons.

REAL MOTIVATION COMES FROM STUDENT SUCCESS - NOT PRETTY TEXTS

THE MAJOR OBSTACLE TO MAKING CHANGES IN MATH COURSES AT THE COLLEGE OR UNIVERSITY LEVEL IS THE TEXTBOOK SELECTION COMMITTEE

The textbook selection committee is itself a collection of instructors, each, with their own individual criteria. I have been a member of many text selection committees, as probably you have. I am as guilty as anyone with my individual hidden criteria. I have said, "I like this text because it presents the topics the way I teach them" -- what I really meant was -- "I like this text because I won't have to spend much time creating a new set of class notes." I have said, "I like this text because it has all these new topics I think we should teach." -- what I really meant was -- "I like this text because I want to learn more of these new topics and I always learn when I teach new topics." We all have our hidden criteria. In contrast: I have never said, nor have I ever heard another person on the committee say "This text presents the material just the way I teach it and that approach is not working no matter how hard I try." I have never said or heard "This text has many new topics, but the problem I see is that the authors did not leave out any of the old topics; how can students learn all of this material in one semester?" Probably nothing is more detrimental to educational progress in colleges and universities than the procedure we use to select textbooks. By its very nature it guarantees the existence of "Status Quo" -- no change, there are always instructors on committees that will vote for no change or at least for very little change -- and say what you may, textbooks really do dictate course curriculum.

As a former editor of several leading mathematics publishers, I was often admonished by teachers, instructors, and professors asking: "Just when is the publishing industry going to give us something different -- a new type of text?"

The answer is simple; **when educators will adopt and buy them.**

The changes in Traditional Developmental Mathematics Texts the last 30 years have been limited to: reducing explanations, increasing the number of examples, and introducing new frill topics. In the last 30 years the textbooks on the "best selling list" for the Developmental Mathematics Courses have NOT introduced a single new approach to teaching mathematics nor reduced the amount of content a text contains. We operate under the illusion that coverage is more important than depth -- that quantity is more important than quality.

At the end of a semester when asked by another instructor I have said and I have often heard others say, **"Well at least I covered all the material."** Recently I was reading an article and it contained a reference to this usage of the word "cover" by educators. The article stated that this usage of cover came from a Latin root that meant "to hide". I was amused to think that when I said "Well at least I covered all the material" I may well have meant "Well at least I hid all the material". Perhaps in our rush to "cover" all the material we may well "hide" the math concepts from developmental students.

If you don't know where you are going, any place will do.

Alice in Wonderland

A NATION AT RISK

What the successful reactors to A Nation at Risk have universally recognized is that the deterioration in the quality of American schooling, signaled by the precipitous decline in average annual SAT scores, was due in no small measure to an over reliance on what John Goodlad has called 'frontal teaching', too much trying to cram facts and information into students' heads and not giving them enough time to understand what they are learning." He goes on to state: "The remedy in the words of Ted Sizer (Leader of the successful Coalition of Essential Schools) is 'less is better'; that is, teach fewer facts and allow more time for discussing and thinking about them.

Dr. George Hanford, President Emeritus of the College Board

Section 3 Reading and Curriculum

THE IMPORTANCE OF READING AND VOCABULARY

Reading It is impossible to expand or verify ideas without the ability to read.

IN TODAY'S WORLD OF TECHNOLOGY, THE HALF-LIFE OF INFORMATION HAS BEEN SET FROM THREE TO SIX YEARS.

STUDENTS WHO CANNOT READ AND UNDERSTAND WHAT THEY READ ARE DOOMED TO BE PASSED BY.

In order to secure and maintain a good paying job in a technological society (This is the main reason students enroll and attend a college or university.) it is imperative to be able to read, including mathematics. Despite this critical need, one-half of the population (40 million adults) are functionally illiterate.

USA Today 9/93

A large percent of high school graduates cannot read; this includes many college capable students who will apply and attend a college or university. The problem is caused by 85% or more of the public schools in America using a "sight word approach" to teach reading. In this approach the commonly used words for each grade level are memorized; increasing the number of words memorized at each grade level. Even if a student memorizes all of the "commonly used words" they only know a percent of the entire English vocabulary. Unfortunately this approach to teaching reading produces more nonreaders than readers. We recommend abandoning the sight word approach to teaching reading and return to a phonics approach.

US Department of Education, 1995

Recent research has indicated that high school or older nonreading students can learn to read within 3 - 4 weeks with phonics training. If the reading problem is not corrected, it is estimated that two-thirds of the American adult population will be functionally illiterate by the year 2000.

Jennie Eller, TV Interview, PBS, April 1996

It is ironic that the same underlying element is at the root of both math and reading remedial problems; over stress on memory. Students cannot memorize everything.

It is imperative that students learn to read technical and reference material, particularly mathematics. Texts must be written at the appropriate reading comprehension level and contain explanations of concepts that can be understood by students.

Today the amount of new reading material and the rate at which it is produced in business and industry is unbelievable, no one has the time to spoon-feed employees as we often do students in education; employees must be able to read technical material which includes math.

Reading We must stop spoon-feeding every topic. Students must be taught that they are responsible for reading and understanding (asking questions if necessary) for some of the material in a course. It is imperative that tests include all topics, even those left for students to read.

Writing We must allow students an opportunity to express their own written thoughts about mathematics. Some students do not find it easy to express their thoughts orally, and writing gives these students an additional opportunity to communicate their ideas to instructors. All students need practice expressing ideas in written form - a necessary ability for future success. It is impossible to communicate ideas without the ability to write. In higher paying positions one must be able to write memos, proposals, responses to other employees or to an employer, and letters of all kinds.

"... writing assignments (Intermediate Algebra) helped remedial students to think mathematically."
Ganguli, Aparna, *Writing to Learn Mathematics: Enhancement of Mathematical Understanding*, The AMATYC Review, Fall 1994

Speaking We still live in a "Pygmalion World", our intelligence or educational background is judged by others as soon as we open our mouths. The *"perceived quality"* of the ideas of our graduates will be judged by their ability to speak English correctly. Students who do not find it easy to express their thoughts orally, need practice expressing mathematics in oral form.

It is essential that every department in a college or university teach reading, writing, speaking, (including common technical terms). Everyone in an institute must take the responsibility of making sure their graduates can read, write, speak, and know modern technical terms -- the basic objectives of education.

IF IT IS NOT POSSIBLE FOR GRADUATES TO COMMUNICATE WITH OTHERS IN THE WORK FORCE, THEN THEIR IDEAS AND CONTRIBUTIONS WILL GO UNKNOWN.

FOR CREATIVITY AND SOLUTIONS TO EVOLVE, THOSE INVOLVED MUST BE BE ABLE TO COMMUNICATE WITH ONE ANOTHER.

STUDENTS WITH LOW READING ABILITIES

Students with very low reading ability should spend all of their first energies in college improving their reading skills. Since college and university education is so dependent on reading ability the benefits of this course of action to their eventual success, not only in obtaining a degree, but life, in general, is great. It is suggested that the programs of these students be limited to a remedial reading course and one other elective course (NOT to include a developmental math course).

NEED FOR A THRESHOLD

Probably the least understood aspect of math education is the need for a threshold in a developmental program. What is to be done with students who do not meet minimum reading standards - - students who do not have the ability to read and pass courses. More specifically, what is to be done with students who have a third or fifth grade reading comprehension? Can we reasonably expect a student with a very low reading level to pass a developmental mathematics course in a college or university? It is unfortunate that some colleges and universities have students with very low reading abilities. The problem of reading level is magnified when mathematical terminology is taught in developmental courses; including math terminology greatly increases the reading level. Students in developmental courses need to read mathematics, and understand lectures and discussions about math where mathematical terminology is involved. CSBM established a minimum reading requirement of 8th grade comprehension level for developmental students entering Beginning Algebra. The reason for a reading requirement in math is that the amount of material to be learned within a time span of one semester demands that a student be able to read.

Reading is also important because everyone forgets material that is not constantly used. Within a short period of time, students forget most of what they learned in college. This is particularly true of material not related to their interest. If a student can read and if the student understood the material when it was first learned, then a student can reconstruct concepts after leaving the classroom by reading references as needed. In the CSBM research, we called this skill "reconstructability". If a student cannot read, then what the student learned is probably lost. With few exceptions, developmental students use their math text only as a source of exercises for homework, and many do not even buy a text thinking it is not necessary to have one.

Mathematics Vocabulary

Blum's research (1977), shown below, focused the need for including vocabulary instruction in Developmental Math, and he determined the percent of common math terms the entering students understood. He started by defining math terms; he concluded that there are three types of terms:

Type 1:	A Math Term:	addition, number
Type 2:	An English Word With a Special Math Meaning:	set, element, function
Type 3:	A Group of Words With a Single Math Meaning:	greater than or equal to

Math Words Made Up 23% of the Math Texts used in the courses of entering students prior to coming to the university.

RESULT OF BLUM'S RESEARCH

60% of Math Vocabulary Not Known by	50% of Entering Students
80% of Math Notation Not Known by	50% of Entering Students

... There are different types of reading. "Reading a math text is not like reading a novel or even a history text. Most math students will need help to learn that often every sentence in a math text contains an important piece of information. ..."

Dr. Bill Daggett (Director of the International Institute of Education)

An error frequently made in developmental courses is not expecting students to take more responsibility for their education. It is certainly easier to "spoon-feed" students than to demand and expect them to put forth the effort to learn. It is easier to lecture from prepared lesson notes than to expect developmental students to read their math textbook to learn and ask questions about "what they do not understand". It is this exchange between students and between student and instructor that leads to learning and understanding math concepts.

For the CRITICAL THINKING APPROACH TO TEACHING MATHEMATICS the **"Exercises for Vocabulary"** were developed to foster reading and to aid in the development and understanding of mathematics vocabulary and notation. These exercises were suggested by the *International Reading Association* and their help in establishing a purpose and style is appreciated. The **Exercises for Vocabulary** provide two reading helps. They emphasize to the student that it is important to read every sentence in a math text carefully and they provide help in understanding the new math vocabulary presented in that Unit. The answer for each question of the Examples that follow is italicized and underlined for your convenience.

EXAMPLES FOF EXERCISES FOR VOCABULARY

D1

1. When we write $3+3+3+3 = 4 \cdot 3$ we are using an interpretation of multiplication called *repeated addition*.
2. Repeated addition can also be used to interpret multiplication when one of the factors is a *negative integer*.
3. (positive integer) • (negative integer) = (*negative integer*).

D2

4. Since the order in which two numbers are multiplied does not change the product, a positive integer times a negative integer is the same as a (*negative integer*) times a (*positive integer*).
5. To compute $^-6$ times 3 we can *reverse* the order of the factors and determine the product 3 times $^-6$.
6. We (*can*) use the regular multiplication tables to determine products involving negative integers.

D3

7. Negative *integer* times a positive integer is another way to represent the opposite of that positive integer.
8. It is logical to assume that ($^-1$) times a negative integer represents the *opposite* of that negative integer.
9. One type of difficulty that students have with mathematics is that of not thinking of more than *one way* to interpret symbols.

Exercises 1 and 2 are examples of questions from two consecutive sentences.

Exercise 3 is an example of a question where students must extend the knowledge of their reading to answer the question; that is, while the concept is explained in the text, the answer is not given directly. Students must use their knowledge of the concept to create an answer.

Exercise 8 is an example of a question involving symbols or notation.

BEST COPY AVAILABLE

It has been our experience that the greatest benefit derived from the **Exercises for Vocabulary** is when they are assigned in advance of the lecture for that topic. In other words, the **Exercises for Vocabulary** for a particular Unit would be turned in by the student with the regular Drill and Practice homework from the preceding Unit. This advance assignment indicates that the students have read and at least are aware of the concepts and vocabulary prior to your lecture.

ACCELERATION

There is an aspect of learning that we humans frequently overlook -- maturity. We often rush young children to try things before they are mature enough, sometimes resulting in injury or worse. There has been a great amount of data gathered on the social problems generated for young students that are accelerated into a university environment before they are mature enough to handle the social aspects of college life. While these students are academically qualified, they do not have the social skills needed. Recently, this problem has resurfaced with the arrest of a Unabomber suspect; it has been suggested that he became so withdrawn and anti-social because he entered the university too young, before he had developed the necessary social skills to cope.

Are we obsessed with acceleration?

In mathematics education we rush 3rd and 4th graders into fractions before they have the math maturity to understand fractions. Have you ever wondered why it is necessary to teach and reteach and reteach fractions at every grade level including college? After all, in the late 1700's students went to University to study fractions.

Are we obsessed with acceleration?

Why do we rush developmental math students into functions before they have the math maturity to understand these concepts? Maturity that comes from first understanding the concepts of Beginning and Intermediate Algebra. Sure they know functional notation, but we know from test results the developmental mathematics student's real understanding of a function is nill or nonexistent. To teach real understandings of the function concept takes almost a full semester at this level.

Are we obsessed with acceleration?

If a developmental student needs to learn the math concepts from arithmetic to college algebra, as many do; then they will have to learn at least 1000 concepts. This number includes only the most basic concepts needed; not the much higher number (1500 - 1800) usually presented in a traditional text. The CSBM research determined that a developmental student can successfully learn and understand less than half that number in one semester. The typical course now offered in colleges and universities covers all 1000 concepts and more in one 3 hour semester course.

Are we obsessed with acceleration?
Excessive Acceleration causes more dependence on Rote Memory.

INSANITY IS DEFINED AS
DOING THE SAME THING
OVER AND OVER,

EXPECTING DIFFERENT RESULTS.

IS THERE ROOM IN THE PRESENT CURRICULUM FOR CRITICAL THINKING OR ANY NEW ESSENTIAL TOPICS?

In a word, **NO!**

In the present traditional courses there isn't room for any new essential topics.
IF CRITICAL THINKING IS TO BE TAUGHT, THEN TIME HAS TO BE PROVIDED FOR THIS PURPOSE.

The time needed to cover the topics in most existing curricula exceeds the amount of time available. Since the curriculum contained in traditional texts contains more material than instructors and students can complete in one term, there is not time to teach critical thinking or any other essential new topic without an overhaul of the present curriculum. To make room for teaching critical thinking CSBM developed a new 30 unit curriculum. While the 30 unit curriculum contains only 70% to 80% of the amount of material presented in traditional texts it contains all math concepts from arithmetic through College Algebra that are needed to successfully complete a calculus course.

The CSBM research indicated that there is a limit to the number of new concepts which can be introduced successfully and mastered in a single term by developmental students. We found that a student's ability varies with the general level of their motivation, and understanding of the background material mastered in previous courses. If the motivation and background is high, then the number of new concepts that can be presented in a semester is probably near 300; if the motivation and background is low, then the number of new concepts which can be introduced is probably less than 200. These numbers are based on a class meeting 4 times a week for one hour. The 30 Unit curriculum limits the number of new concepts presented in each Unit so that topics can be covered and understood by students. Traditional texts for a one semester Developmental Math course contain over 1500 concepts.

New research by **Richard Paul (1991)** has placed this low mark as 50 new concepts per term. (However, in reading Paul's paper I feel this discrepancy is caused by the method of how "new concepts" are tabulated; CSBM counted each new major concept and each new subconcept of that major concept, I think Paul counted only the major concepts.)

It is also a terrible waste of time to review material, at the beginning of each course, because students have sketchy backgrounds of previous topics. The 30 Unit curriculum does not contain unnecessary reviews; it also has some topics in an order different from the traditional curriculum and the priority of some topics has changed; causing some topics to receive less emphasis, and others to be postponed until later in the curriculum.

In traditional texts topics were included to "make the text more adoptable" by appealing to a greater variety of instructors and, as such, many topics do not fall into the set of topics that are really necessary. The "Standards" of NCTM and the "60% and teach it better" of MAA are only two of the professional articles to emphasize that we must reduce the amount of curriculum. Today, phrases such as: "Quality not quantity" or "Depth not breadth" permeate the professional writings related to curriculum and critical thinking.

CSBM 30 Unit Curriculum

COMPETENCY ARITHMETIC

Unit 1 NUMBERS AND NOTATION
Unit 2 DECIMALS
Unit 3 FRACTIONS
Unit 4 PERCENT

GEOMETRIC & MEASUREMENT TOPICS

Unit 5 GEOMETRY
Unit 6 METRIC MEASUREMENT
Unit 7 LOGIC
Unit 8 DATA MEASUREMENT

BEGINNING ALGEBRA

Unit 9 INTEGERS AND OPERATIONS WITH INTEGERS
Unit 10 BASIC NUMBER THEORY
Unit 11 RATIONAL NUMBERS
Unit 12 ALGEBRAIC EXPRESSIONS AND EXPONENTS
Unit 13 POLYNOMIALS
Unit 14 LINEAR EQUATIONS AND INEQUALITIES
Unit 15 LINEAR SYSTEMS

INTERMEDIATE ALGEBRA

Unit 16 POLYNOMIAL OPERATIONS
Unit 17 RADICALS
Unit 18 FACTORING QUADRATIC POLYNOMIALS
Unit 19 SOLVING QUADRATIC EQUATIONS
Unit 20 RATIONAL EXPONENTS
Unit 21 RATIONAL EXPRESSIONS
Unit 22 COMPLEX NUMBERS

COLLEGE ALGEBRA

Unit 23 LINEAR FUNCTIONS AND INEQUALITIES
Unit 24 QUADRATIC FUNCTIONS AND INEQUALITIES
Unit 25 POLYNOMIAL AND RATIONAL FUNCTIONS
Unit 26 EXPONENTIAL AND LOGARITHMIC FUNCTIONS
Unit 27 PERMUTATIONS, COMBINATIONS, BINOMIAL THEOREM, & MATHEMATICAL INDUCTION
Unit 28 ALGEBRA OF ORDERED PAIRS

OR PRE-CALCULUS

Unit 23 -- Unit 26 Same as the Units in College Algebra

Unit 23 LINEAR FUNCTIONS AND INEQUALITIES
Unit 24 QUADRATIC FUNCTIONS AND INEQUALITIES
Unit 25 POLYNOMIAL AND RATIONAL FUNCTIONS
Unit 26 EXPONENTIAL AND LOGARITHMIC FUNCTIONS
Unit 27 TRIANGULAR TRIGONOMETRY
Unit 28 FORMULAS AND IDENTITIES
Unit 29 GRAPHS OF CIRCULAR FUNCTIONS
Unit 30 APPLICATIONS OF THE CIRCULAR FUNCTIONS

Probably the greatest amount of resistance to the CSBM research results, or the materials that have come from this research, relates to deletions and changes in the curriculum; or the slower pace of the CSBM curriculum. I am constantly surprized by instructors who defend the teaching of the procedure for taking a square root, the procedure for interpolations, and so on, as vital to the curriculum for Developmental Math. I am also surprized at those who are frustrated with their results when they attempt to teach more than 1000 math concepts in a 3-hour semester course. When I began discussing changes or pacing in the curriculum suggested by the CSBM research, I quickly found that I was tampering with "Sacred Cows". Every instructor has favorite topics or topics for which they have developed special presentations, and one dare not suggest deletions or changes in that area; and because developmental math is, after all, a review of high school math, many instructors feel that there is no limit to the number of concepts that can be taught and learned in one semester.

"... you may see the need for curriculum reform as obvious, you should probably not expect all members of your department to agree enthusiastically, if at all. Too many mathematicians view their own training, or lack of, as the only way to teach mathematics. Such individuals can always be counted on to oppose curriculum change.

They dismiss observations that students a) do poorly in existing courses, b) do not appreciate the mathematics they are being taught, c) cannot transfer their mathematics to other disciplines, and d) retain frightening little of the material they were previously taught as either -- undocumented conjecture or something that is solely the fault of the students. **It cannot be the course or the way it was presented.** ..."

Sheldon P. Gordon, *Implementing Change in the Mathematics Curriculum*, The AMATYC Review, Fall 1994

SHORT-TERM MEMORY STORED IN "MAILBOXES" - - NO TRANSFER

The analogy of student learning to office mailbox stuffing was given by some researchers at San Jose State University. Their research indicated that students memorized facts and placed them into their **short term memory** in a fashion similar to stuffing personal mailboxes in an office. As such, these facts were unrelated and often the same fact in a different context would be stuffed into a separate mailbox within the same period of time; then, usually the end of the semester for most students, but after each test for others, the mailboxes would be dumped and stand empty, ready for the next stuffing. Obviously the interrelation of some of the facts, the understanding of most, and long term memory does not exist with mailbox stuffing.

When too much material is presented in too short of time, students are forced to memorize and stuff this information into their short term mailboxes.

WE MUST SET PRIORITIES AND DETERMINE WHAT IS IMPORTANT IN THE TEACHING OF DEVELOPMENTAL MATHEMATICS AND THE AMOUNT OF CONTENT IN TEXTS MUST BE REDUCED AND REVISED.

Section 4 Introduction to Math Critical Thinking

WHAT IS CRITICAL THINKING ??

The human animal is the only animal that has a high enough order of intelligence that they are able to give meaning to Ideas, Concepts, Models, Theories, & Explanations;

the only animal whose thinking can be characterized as Clear, Precise, Accurate, Relevant, Consistent, Profound, & Fair.

WHAT IS CRITICAL THINKING ??

The human animal is the only animal that has a high enough order of intelligence that they are able to give meaning to Contradictions, Deceptions, Misconceptions, Distortions, and Stereotypes;

the only animal whose thinking can be characterized as Imprecise, Vague, Inaccurate, Irrelevant, Superficial, Trivial, and Biased.

THE LOGICAL ILLOGICAL ANIMAL Richard Paul

CRITICAL THINKING

Everyone thinks at some level, it is our nature to do so; however, undirected thinking is often biased, distorted, nonproductive, illogical or partially uninformed. Because our students are able to use undirected thinking they feel they know all there is to know about thinking. Our challenge is to demonstrate to our students that they can extend their thinking abilities to include educational critical thinking. They must do enough thinking to question, analyze, and extend data and concepts that affect their lives. Some of the CEO's in industry estimate that 90% of their future employees will be needed as thinkers, expediters, and communicators; to be able do this successfully students will have to read, write, and think. This ability will need to include math.

DRILL AND PRACTICE - A NECESSARY COMPONENT FOR LEARNING

Before we continue investigating critical thinking, it is important to mention the place of drill and practice. In the CSBM research we found that many students must know the "how" of something before they are ready to understand the "why" of the concept involved. Nonmathematical students need "drill and practice" exercises as part of their learning activities in mathematics. While the CSBM researchers concluded that drill and practice is a necessary part of learning -- in developmental mathematics it is very easy to "over kill". In this respect an effort was made in the CSBM materials to limit the number of repetitive exercises and to try to increase the number of exercises that caused the students to think about using algorithms in various ways.

Nonmathematical students need "drill and practice" exercises as part of their learning activities in mathematics. The CRITICAL THINKING APPROACH TO MATHEMATICS materials contain *Exercises for Computation* which are similar to the types of exercises found in traditional texts. These materials also contain *Self Tests* for students to check on their progress.

FROM A STUDENT'S POINT OF VIEW

ALGORITHM CREATED BY 4th GRADERS

In math research we often attack a problem by asking 3 questions: Can I do this problem? Can I change it to one I do know how to work? Will I have to create new math?

When a 4th grade class had learned their division facts. Their teacher asked the students to see if they could extend their knowledge to divide a four-digit dividend by a single-digit divisor. Two of the boys spent the weekend creating an algorithm to do this. In terms of math research, they changed the given problem to one they could work; dividing a two-digit number by a single-digit. Keeping track of each place value, they developed the algorithm shown below.

Can You find a way to do this? $8\overline{)23563}$

2000	7000	2000
8 ⟌ 23000	563	
1600	7563	
7000		
900	300	900
8 ⟌ 7500	63	
7200	363	
300		
40	40	40
8 ⟌ 360	3	
320	43	
40		
5		5
8 ⟌ 43		
40		
3	ANSWER	2945 R 3

IS THIS ANSWER CORRECT? IS THIS ALGORITHM CORRECT?

IF YOU WERE TH ESE STUDENTS 4th GRADE TEACHER, WHAT WOULD YOUR REACTION BE?

They proudly went to school to show the teacher their algorithm; what is a typical reaction of a teacher when students come up with an alternate algorithm? Sad, but You are right, because it was not the long division algorithm we normally use, the teacher told the boys their algorithm was wrong.

Why are we educators so entrenched?

FROM A STUDENT'S POINT OF VIEW

ANOTHER ALTERNATE ALGORITHM

Consider

$$\frac{4}{5} * \frac{3}{4} = \frac{12}{20} \qquad \frac{9}{7} * \frac{2}{9} = \frac{18}{63}$$

$$\frac{12}{20} \div \frac{4}{5} = \frac{3}{4} \qquad \frac{18}{63} \div \frac{2}{9} = \frac{9}{7}$$

Based on the algorithm for the multiplication of fractions an alternate algorithm for the division of fractions is possible. Students will say: "How did you get the correct answer when you did not invert and multiply" Why does this algorithm work? Will this algorithm always work?

NOTATION PROBLEMS

Consider 13^2

$13^2 = 169$

But $13 = 10 + 3$

What is wrong with

$10 + 3^2 = 19?$

Thus $13^2 = 19$

What is x^2

If $x = 13$?

What do you say to a student who who thinks that 13^2 should be 19? What reasons do we have in math that 13^2 must be 169? Is it PMDAS ?

Consider $\frac{3^2}{4}$

This $\frac{3^2}{4} \stackrel{?}{=} \frac{9}{16}$

or this $\frac{3^2}{4} \stackrel{?}{=} \frac{9}{4}$

What is x^2

If $x = 3/4$?

In this example we have again placed a raised 2 after the 3; the same as in the example above. Why in this example is only the 3 squared and not the 4? What about PMDAS? Are we consistent in math notation? Can we use the statement "That's just how we always do it" as a reason for students? Is it any wonder that students often feel we mathmaticians are not critical thinkers? In CSBM we found that the understood grouping symbols in math notation cause many problems for developmental students.

FROM A STUDENT'S POINT OF VIEW

ALTERNATE NOTATION

Nonmathematical students are often confused by the double use of the minus sign; that is, the minus sign is used to represent both subtraction and a negative number. Are both – 3 and $^{-}3$, correct? YES! Correctly defined, $^{-}3^{2} = +9$. In the **CSBM** Beginning Algebra text, $^{-}3$ is used because it separates the two concepts of subtraction and negative numbers allowing for easier learning and more understanding by Developmental Math students. Also many students have used this notation for negative numbers in high school and already understand the notation.

Consider -3^2

$-3^2 \stackrel{?}{=} 9$

$-3^2 \stackrel{?}{=} -9$

Consider $^{-}3^2$

$^{-}3^2 \stackrel{?}{=} 9$

$^{-}3^2 \stackrel{?}{=} -9$

CSBM defined $^{-}3$ to be (-3)

and ^{-}x to be $(^{-}1)\,x$

What is -3 squared on a calculator?

What is x^2 If $x = {}^{-}3$?

Some college instructors have objected to this notation in the **CSBM** Beginning Algebra, some even tell me it is mathematically wrong. Why are we educators so entrenched? The need for a separate notation for negative numbers declines drastically toward the end of Beginning Algebra course; therefore, starting with the **CSBM** Intermediate Algebra, the notation $^{-}3$ is dropped and – 3 is used in the rare occasions when a specific notation for a negative number is needed. This gradual switch from $^{-}3$ to – 3 did not cause a problem for students while allowing the two concepts to be taught separately.

Facts you have probably learned from your students

The problem with notation and grouping arises in a classroom in many ways; sometimes allowing us to have fun with our students. In the examples below, students often think it is possible to cancel the x's and the 4's. If this happens, then using consistEncy in notations and algorithms many interesting facts are possible; such as the last example.

$$\frac{x \pm 5}{x} = 6 \qquad \frac{4 \pm 4\sqrt{3}}{8} = \frac{1 \pm 4\sqrt{3}}{2}$$

Because $13 = 4 + 9$

$$\frac{13}{4} = \frac{4 + 9}{4} = 10$$

WHAT IS CRITICAL THINKING ? SOME DEFINITIONS

Foundation for Critical Thinking Sonoma State University, *Workshop on Critical Thinking Instructional Strategies*, Denver, CO.; 1993

"Critical Thinking is that mode of thinking -- about any subject, content, or problem -- in which the thinker improves the quality of his or her thinking by skillfully taking charge of the structures inherent in thinking and imposing intellectual standards upon them."

Richard Paul Sonoma State University Rohnert Park, CA 1992

"Critical thinking is disciplined, self-directed thinking that exemplifies the perfections of thinking appropriate to a particular mode or domain of thought. " Paul continues stating that critical thinking can be perfected to be a strong positive influence - or - critical thinking can just happen and become a strong negative influence.

R. H. Ennis Harvard Educational Review , 1962 # 32

"A concept of critical thinking must include a correct assessment of statements."

R. H. Ennis *Philosophy and Education* , 80th Yearbook, National Society for the Study of Education,1981

"Critical thinking is reasonable, reflective thinking that is focused on deciding what to believe or do."

E. M. Glaser *Critical Thinking: Educating for Responsible Citizenship in a Democracy,* Phi Kappa Phi Journal, 1985 # 65

"Critical Thinking involves three principal elements: attitude of being disposed to consider in a thoughtful, perceptive manner the problems and subjects that come within the range of one's experience, knowledge of the methods of logical inquiry and reasoning, and skill in applying these methods."

J. Chaffee *Teaching Critical Thinking Across the Curriculum,* ISDA Journal 1987 # 3

"Critical Thinking is making sense of our world."

M. L. Poage *Critical Thinking Approach to Mathematics Texts* 1992

"The ability and willingness to ask questions and to seek answers toward the positive solution of a problem or situation which demands thinking that is self-directed, and self-corrective."

CHARACTERISTICS OF CRITICAL THINKING

THE FOUNDATION FOR CRITICAL THINKING, Santa Rosa, California

A UNIQUE KIND OF PURPOSEFUL THINKING	Any subject area requires intellectual fitness training for the mind.
IN WHICH THE THINKER SYSTEMATICALLY AND HABITUALLY	Actively develops traits such as intellectual integrity, intellectual humility, intellectual empathy, fairmindedness, and intellectual courage.
IMPOSES CRITERIA AND UPON INTELLECTUAL STANDARDS THE THINKING	Identifies the criteria of reasoning (precision, depth, accuracy, and sufficiency) and establishes a standard of thinking by which the effectiveness of the thinking will be assessed.
TAKING CHARGE OF THE CONSTRUCTION OF THINKING	Awareness of the elements of thought (assumptions and points of view) that are part of well-reasoned thinking; a conscious, active and disciplined effort to address each element.
GUIDING THE CONSTRUCTION OF THE THINKING ACCORDING TO THE STANDARDS	Continually assessing the course of construction during the process, adjusting, adapting, improving, using the criteria and standards.
ASSESSING THE EFFECTIVENESS OF THE THINKING ACCORDING TO THE PURPOSE, THE CRITERIA, AND THE STANDARDS	Deliberately assessing the thinking to determine its strengths and limitations and studying the implications for further thinking and improvement.

EDUCATIONAL CRITICAL THINKING

Educational Critical Thinking is logical and productive thinking that is capable of extending ideas, creating new ideas, and synthesizing patterns of thought. Educational Critical Thinking is a type of disciplined reasoning. Educational Critical Thinking must be self-directed, self-disciplined, self-monitored, and self-corrective.

It is now generally conceded that the art of thinking critically is a major missing link in education today, and that effective communication and problem solving skills, as well as mastery of content require critical thinking. It is also generally recognized that the ability to think critically becomes more and more important to success in life as the pace of change continues to accelerate and as complexity and interdependence continue to intensify. It is also generally conceded that some major changes in instruction will have to take place to shift the overarching emphasis of instruction from rote memorization to effective critical thinking (as the primary tool of learning).

Richard Paul 1995

The mind has three basic functions: thinking, feeling, and wanting. The process of thinking creates meaning (making sense of events of our lives thereby). The process of feeling monitors those meanings (evaluation how positive and negative the events of our lives are, given the meaning we ascribe to them). The process of wanting drives us to act (in keeping with our definitions of what is desirable and possible); there is an interrelation between thinking, feeling, and wanting. When students think a subject they are required to study has no relationship to their lives and values, they feel bored by instruction in it, and develop a negative motivation with respect to it.

Instruction that fails to address the affective side of students' lives can eventually turn students into inveterate "enemies" of education. For example, students who are force-fed mathematics in a way that ignores student emotions typically end up with a bad case of "math hatred"; they avoid anything mathematical and they view math as unintelligible, "just a bunch of formulas," unrelated to anything important in their lives.

HOW DO WE CHANGE INSTRUCTION TO EMPHASIZE CRITICAL THINKING?

LEARNING IS A VERY HUMAN THING -- IT INVOLVES STUDENTS -- IT IS A TWO-WAY STREET WHICH INSTRUCTORS MUST WALK WITH THEIR STUDENTS.

INFORMATION INPUT

A very interesting research was being done by Carnegie Institute of Technology for handicapped students. The research concerned information input rates of students.

TYPE	NORMAL	SPEED
READING RATE Average College Student	250 wpm	800 wpm
TALKING RATE Lecturer Teacher TV Announcer	120 wpm	400 wpm

Based on their research, they concluded that the average college student's brain input was at least 235 - wpm; about twice the speed of a lecturer. This is the main reason that students become passive in a lecture -- **we talk too slow**. They also found that, within limits, a student's attention to a lecture varied with the rate of speaking; the faster a lecturer talked, the greater the percent of student attention.

Lecture We must limit lecture time to 20 - 30 minutes. Lectures which exceed 30 minutes in a typical class are not an effective strategy for learning. Research indicates that the attention span of most all students expires after 25 - 30 minutes, particularly in a course which isn't in their special field of interest.

"Speak less so they can think more! (Try to limit the lecture to not more than 50% of total class time.) ... Think aloud in front of your students. Let them hear you thinking; better, puzzling your way slowly through a problem . Very significant consequences follow from how students learn. The depth with which they understand a concept is in direct proportion to the degree to which they have engaged in intellectual labor to figure it out for themselves. ..."

Richard Paul, Foundation for Critical Thinking, 1993

In the Critical Thinking Approach the lecture is a necessary component; albeit shortened to 30 minutes to better fit the attention spans of students. Instructors, based on their experience, limited their lecture to those topics in a lesson that were the most difficult. Another important reason to shorten the lecture is to force students to put more reliance on, and to have more confidence in, learning math by reading a text and discussing concepts with other students. The second half of the class period should be a workshop for Critical Thinking Exercises; providing the necessary time for teaching terminology, reading, critical thinking, and problem solving. The advantage of doing the Critical Thinking exercises is the increased student involvement and attention; students become active learners by being involved in open discussions. "Two - way" communication happens through discussions; students should be encouraged and invited to take part in discussions. In CSBM we found that far more learning and understanding resulted when students were involved in give and take discussions with peers as opposed to "passive attention" to lectures by instructors.

Critical Thinking Exercises

CRITICAL THINKING SKILLS CAN BE LEARNED MORE EFFECTIVELY WHEN STUDENTS DO EXERCISES OR PROBLEMS IN GROUPS OF 2 - 4.

"LEARNING IS MORE EFFECTIVE WHEN THE STUDENT ACTIVELY PARTICIPATES IN THE LEARNING EXPERIENCE "

C. C. Cole Jr., *Improving Instruction: Issues and Alternatives for Higher Education*, ERIC, Higher Ed. Report #4, 1982

"When students do not actively think their way to conclusions, when they do not discuss their thinking with other students or their instructor, when they do not entertain a variety of points of view, ..., actively question the meaning and implications of what they learn, ..., they do not achieve higher-order learning. .... As a result, their adaptability, their capacity to learn on the job and in their personal and civic lives, is severely limited."

Richard Paul, ERIC, 1992

We are entering a rapidly changing world. Many experts believe that future graduates will need to change fields of employment as many as four to five times. That is, if graduates have a degree in one field, they may need to change fields in order to continue employment. This changing will place a great responsibility on graduates to be able to adapt and to educate themselves in that new field. Critical thinking skills are essential for meeting these responsibilities.

Section 5 Critical Thinking Approach to Mathematics

HOW ARE COURSE MATERIALS CHANGED TO EMPHASIZE CRITICAL THINKING?

Text materials must include appropriate types of exercises to foster critical thinking; exercises designed specifically to encourage "active learning". Active learning involves groups of students working together on an exercise. There are several types of exercises that can be used to accomplish this purpose:

discussion, explanation, discovery exercises, and problems for problem solving.

The CRITICAL THINKING APPROACH TO MATHEMATICS materials contain the discussion, explanation, and discovery exercises in the *Exercises for Understanding* and, in addition, *Problems for Problem Solving* also included. Discussion and Explanation Exercises are straight forward and fairly easy to create; just use the terms "DISCUSS" or "EXPLAIN" in a question or exercise.

Discussion Exercises

Discussion exercises involve more than one student. The purpose of such exercises is to allow for a free exchange of ideas among students (occasionally the instructor will be involved). Discussion exercises can be a group or class activity, or be used as written exercises. Students who have not yet understood will often ask questions or make statements which are unrelated to the discussion topic. With encouragement for each attempt, it is important to keep the discussion focused on the topic at hand. The most productive discussions have mostly student input with instructor input limited to necessary redirection.

EXAMPLES OF DISCUSSION EXERCISES

EXERCISES FOR UNDERSTANDING Beginning Algebra

B3

51. If I go into the store with only $10.00 and my purchase plus tax comes to exactly $10.00, what is my financial state at this point ?

B6

56. On a series of 10 downs a football team made the following gains and losses: gained 7 yards, lost 2 yards, gained 5 yards , gained 12 yards, lost 6 yards , gained 4 yards, gained 16 yards, lost 17 yards, lost 6 yards, and gained 12 yards. If they started on their own 20-yard line where are they at the end of these 10 plays ? If you were the coach, what play would you call next ?

F2

75. Discuss if and when the following statement is true: $|a| = |^{-}a|$.
76. Discuss if and when the following statement is true: $|b - a| = |a - b|$.

Explanation Exercises

Explanation exercises can involve a single or a group of students. The purpose of such exercises is to allow students to communicate their knowledge of the given topic. After an explanation is completed, other students can add their comments.

EXAMPLES OF EXPLANATION EXERCISES

EXERCISES FOR UNDERSTANDING Beginning Algebra

B5

53. If x represents an integer, explain why it would be correct to change $x + 0$ to $x + (9 + {}^{-}9)$?

F1

71. Explain the answer to $|0|$.
72. Explain how the idea of absolute value was used to compute the distance between two coordinates.

F2

74. Explain how ^{-}m may represent a positive number.

BEST COPY AVAILABLE

A group of students can volunteer answers to Discussion or Explanation Exercises. Instructors should call on students who seldom or never volunteer. Make sure that students supply reasons with their answers. After the first group completes their answers, call on a second group to analyze, summarize, or disagree with the first group's answers. This will stimulate more active listeners.

Discovery Exercises

Unfortunately it is not as easy to create DISCOVERY EXERCISES. Discovery exercises also involve the whole class or a class group. The purpose of such exercises is to create positive learning attitudes and help students to gain confidence through a group investigative approach to problem solving; this approach is often referred to as "**active learning**". It is suggested that Discovery Exercises be done in groups of 2 - 4 students and that the instructor act as a moderator, providing help as needed.

EXAMPLE OF DISCOVERY EXERCISE

EXERCISES FOR UNDERSTANDING **Beginning Algebra**

Use Exactly 5 fours 4, 4, 4, 4, 4

as many arithmetic operations as needed + − • +

and as many grouping symbols as needed ( ) [ ]

To create expressions for 0, 1, 2, 3, ..., 18, 19, 20

FOR EXAMPLE

Write five 4's horizontallty with space in between 4 4 4 4 4

Insert the arithmetic operations 4 + 4 • 4 + 4 + 4

Finally, insert the grouping [(4 + 4) • 4 + 4] + 4 = 9

Remember you need five 4's in each expression. It should be noted that classroom noise level is directly proportional with success in discovery.

The expressions for five 4's are not unique; it is not uncommon for two student groups to get different expressions for the same number -- when this happens they will need to know which is correct -- because everyone knows you can have only one correct answer in math.

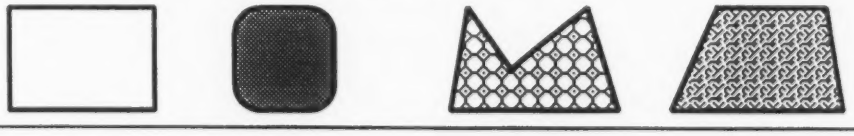
GEOMETRIC EXAMPLE OF A DISCOVERY EXERCISE

Each of the following is a line segment.

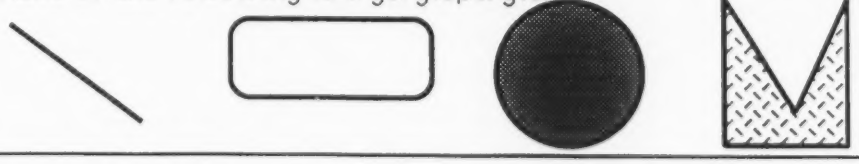
None of the following is a line segment.

Which of the following is a line segment?

Each of the following is a gorgieporgie.



None of the following is a gorgieporgie.



Which of the following is a gorgieporgie?

Example 3 of a **DISCOVERY EXERCISE** from CSBM materials

Using the numbers $^{-}1$ and $^{-}3$, exponents, and grouping symbols as needed, create exponentials to represent each of the following:

1, $^{-}1$, 3, $^{-}3$, 9, $^{-}9$, 27, $^{-}27$, 81, $^{-}81$, 243, and $^{-}243$.

Using the numbers $^{-}1$ and $^{-}3$, exponents, and grouping symbols as needed, create exponentials to represent the reciprocals of each of the following:

1, $^{-}1$, 3, $^{-}3$, 9, $^{-}9$, 27, $^{-}27$, 81, $^{-}8$,1, 243, and $^{-}243$.

Example 4 of a **DISCOVERY EXERCISE** from CSBM materials

Evaluate the following: 1^1, 1^2, 1^3, 1^4, and 1^n where n is any positive integer.

Discuss: Is this notation permissible? Can n be a negative integer? Create a theorem involving $1^n =$ ___.

Evaluate the following: 0^1, 0^2, 0^3, 0^4, and 0^n where n is any positive integer.

Discuss: Is this notation permissible? Can n be a negative integer? Create a theorem involving $0^n =$ ___.

Example 5 of a **DISCOVERY EXERCISE** from CSBM materials

A1-5

64. Terminating decimals are rational numbers. To change a terminating decimal to a common fraction, use the digits of the fraction as the numerator and the place value of the last digit as the denominator. As a final step, reduce if possible.

Study the example then

read, " twenty-five hundredths"

$0.25 = \frac{25}{100} = \frac{1}{4}$

convert the following terminating decimals to common fractions.

(a) 0.75 (b) 0.34 (c) 0.125

65. Repeating decimals are rational numbers. To change a repeating decimal to a common fraction, use the procedure shown below. As a final step, reduce if possible.

To change 0.3 to a fraction: let n be the number of digits in repeating block (1) and

let $x = 0.3333333...$

$n = 1.$

$10^n x = 10x = 3.33333...$

(subtract) $x = 0.33333...$

$9x = 3.0...$

$x = \frac{1}{3}$

To change 0.45 to a fraction: let n be the number of digits in repeating block (2) and

let $x = 0.454545...$

$n = 2.$

$10^n x = 100x = 45.454545...$

(subtract) $x = 0.454545...$

$99x = 45.0...$

$x = \frac{45}{99} = \frac{5}{11}$

Study the examples above, then convert the following repeating decimals to common fractions.

(a) $0.\overline{6}$ (b) $0.\overline{9}$ (c) $0.\overline{27}$

ANSWERS FOR EXAMPLES ABOVE IN THE APPROPRIATE INSTRUCTOR'S MANUAL.

RETURN TO TRADITIONAL TEACHING

FIVE STEP ALGORITHM

EXAMPLE

$$\begin{array}{r} 3427 \\ \underline{1764} \end{array}$$

STEP 1

$$\begin{array}{r} 3427 \\ \underline{1764} \end{array}$$

COPY 1ST LINE

$$\begin{array}{r} 3427 \\ \hline \end{array}$$

STEP 2

$$\begin{array}{r} 3427 \\ \underline{1764} \end{array}$$

DETERMINE 9'S COMPLEMENT OF 2nd LINE

$$\begin{array}{r} 3427 \\ \underline{8235} \end{array}$$

STEP 3

ADD

$$\begin{array}{r} 3427 \\ 8235 \\ \hline 11662 \end{array}$$

STEP 4

REPOSITION FIRST 1

$$\begin{array}{r} 3427 \\ 8235 \\ \hline \boxed{1}1662 \\ \quad\quad\ \llcorner 1 \end{array}$$

STEP 5

RE - ADD

$$\begin{array}{r} 3427 \\ 8235 \\ \hline 1662 \\ \quad\quad 1 \\ \hline 1663 \end{array}$$

FINAL ANSWER 1663

DRILL AND PRACTICE FOR FIVE STEP ALGORITHM

1 2456
<u>1321</u>

2 3972
<u>1673</u>

3 8347
<u>6541</u>

4 4658
<u>1672</u>

5 9325
<u>6147</u>

6 4829
<u>3411</u>

ANSWERS 1 1135 2 2299 3 1806 4 2986 5 3178 6 1418

TEST FOR FIVE STEP ALGORITHM

1 5316
+ 2113

2 9246
+ 6255

3 8719
− 5000

4 2002
− 622

NEED FOR STAFF TRAINING

Probably the most Critical Need when implementing curriculum reform is that of staff training. In order to use the Critical Thinking Approach to Mathematics it is necessary for the staff to understand the changes in both curriculum and pedagogy. The need for training sessions may be particularly essential for those colleges and universities that use adjunct faculty or teaching assistants to cover these courses. I hear more and more about the difficulties encountered in developmental math programs related to adjuncts or teaching assistants (and even some instructors or professors). Many math teachers are so "entrenched" in teaching the way they learned or the way they have "always taught" that it is just not possible for them to even consider a different approach, despite the fact that a different approach could produce a higher rate of student success. There is also a need to provide additional training for lab assistants for these courses.

We math instructors are so *entrenched*, we cannot change our method of instruction easily - if at all.

Dawn Kindall, Newbury College Conference, March 1996

1. 3203, 2. 2991, 3. 3719, 4. 1380

PEER TEACHING

When student success is a high priority, upperclass math students are often overlooked as an efficient resource of instruction. Several recent research studies of peer teaching (from one room schools to programs using peer teaching) indicate higher student success when peer teaching is involved. In the CSBM program we found that upperclass math tutors provided several significant advantages. They were a very cost effective means of increasing individualized or small group instruction. They were generally more flexible, and more willing to listen to our specific directions which resulted in fewer administrative problems. Most importantly, their teaching provided a higher rate of student success. It is difficult not to let our pride get in the way of higher student success (often upperclass students can do a better job than instructors). In the CSBM program a combination of willing professors, instructors, adjuncts, teaching assistants, and peers were used to achieve a high rate of student success.

ONE POSSIBLE ANALYSES OF THE FIVE STEP ALGORITHM

Five Step Algorithm Theory for a 4-digit Number

Denote the digits of the first top number by *abcd*, and the bottom number by *efgh*.

The first step of the algorithm accomplishes no math change.

The second step is to compute the 9's complement in effect creates the difference between 9999 and the bottom number (9999 – *efgh*)

The third step is to add the numbers (*abcd* + 9999 – *efgh*)

The fourth step is to move the one on the left, in the ten thousand's place, to the one's place on the right; this move subtracts 10,000 and adds 1 (– 10,000 + 1 = – 9999)

The fifth step simplifies the expression:
(*abcd* + 9999 – *efgh*) – 10,000 + 1 = *abcd* – *efgh*

The THE FIVE STEP ALGORITHM is a Different Algorithm for Subtraction (Used by calculators).

IN THE LONG RUN

WE HIT ONLY WHAT

WE AIM AT

Section 6 PROBLEMS FOR PROBLEM SOLVING

PROBLEM SOLVING IS AN APPLICATION OF CRITICAL THINKING

Problem Solving is the means by which an individual uses previously acquired knowledge, skills, and understanding or resources to satisfy the demands of an unfamiliar situation. Traditional "word problems" are seldom problems. One of the main factors in real problem solving is the student's "unfamiliarity" with an element of the problem; this is not usually the case with traditional word problems. If critical thinking skills are taught, because of the close relationship between critical thinking and problem solving, then solving a few problems which illustrate problem solving techniques will suffice.

PROBLEM SOLVING IS APPLIED CRITICAL THINKING.

If one is able to think critically, then solving a problem is not a big mystique. Problems that can be used for enhancing problem solving techniques must be more complex than typical "word problems"; problems should not be solvable by merely fitting some known algorithm. More problem solving skills are learned when students and the instructor work on solutions in groups. Estimation must be included as an integral part of problem solving. In the Critical Thinking Approach the problem is completed by the group or class in a series of challenges where students are encouraged to verbalize and discuss their thinking processes.

WORD PROBLEMS VERSUS PROBLEM SOLVING

To determine or compute the answer to a traditional word problem a student is seldom asked to do more than sort out which of the algorithms presented in the chapter or section currently under study applies, and use that algorithm correctly. One of the main factors in real problem solving is that the problem contains at least one element that comes as a complete surprise which the solver has not encountered before. Traditional word problems do not contain that "surprise element".

> Solving a problem means finding a way out of a difficulty, a way around an obstacle, attaining an aim which was not immediately attainable. Solving problems is the specific achievement of intelligence, and intelligence is the specific gift of mankind: solving problems can be regarded as the most characteristically human activity. ...
>
> **George Polya, *Mathematical Discovery* (*On understanding, learning, and teaching problem solving*) *Vol. 1*, 1962**

While working "word problems" is seldom "problem solving," texts should include appropriate word problems when the application of mathematics is honest; students can benefit from discussing solutions and alternate solutions as a prelude to problem solving.

To solve a problem one must attack the problem in some manner. Usually the best problem solvers solve problems by asking questions of themselves about the problem; they also have a set of tactics or heuristics that they use to help them unravel the problem. It is a worthwhile activity to take the time to create a list of tactics with students.

Tactics

Below is a list of some of the usual tactics.

Identify the problem: State the problem in your own words; What is given? What is missing?

Search for relations: Is the problem a known type? Is it related to other problems? Will substituting numbers for the unknowns give a clue?

Delimit the problem: Can the problem be separated into parts? Are there superfluous details?

Search for organization: Can the data be arranged in a table? Can a figure be drawn?

Determining formulas: Can the answer be estimated?

Use of references: Tables, Texts, Class Notes.

Restrictions or limitations: Are there any (numbers) that cause exceptions?

The length or order of a list of tactics is not significant nor can all tactics be applied to every problem.

> Descartes mediated upon a universal method for solving problems. Leibnitz very clearly formulated the idea of a perfect method. yet the quest for a universal perfect method has no more succeeded than did the quest for the philosopher's stone which was supposed to change base metals into gold; there are great dreams that must remain dreams. Nevertheless, such unattainable ideals may influence people: nobody has attained the North Star, but many have found the right way by looking at it.
>
> **George Polya, *Mathematical Discovery* (*On understanding, learning, and teaching problem solving*) *Vol. 1*, 1962**

CRITICAL THINKING and PROBLEM SOLVING

We must teach so that students learn to think critically as they read course material, write about course material, take part in discussions, explain math concepts to other students; in general, in all situations where they exchange ideas with other students and their instructors. This type of thinking and exchanging ideas is the essence of problem solving.

To increase the level of critical thinking involved in problem solving it is suggested that a student or a group of students keep track of their reasoning processes, step by step in writing, as they attempt to solve a problem. As necessary, the reasoning process can be analyzed to determine ways for improvement; refining thinking processes improves the ability to solve problems. Hypothesizing, trial and error, estimation, how to use resources, and reasonableness, as well as flexibile methods, should be encouraged.

EXAMPLE 1: Problems for Problem Solving,
Found In Critical Thinking Approach to Beginning Algebra

Represent each height by an integer, then order from smallest to largest: Mount Everest 29,028 feet; Mount Gasherbrum III 26,090 feet; deepest land depression 1291 feet; altitude of highest airport 13,600 feet; greatest depth by submarine 8310 feet; deepest drilling 30,050 feet.

Challenge 1 Do you need a reference point?

Challenge 2 What reference point will you use?
Will the reference point used make any difference?

Challenge 3 Use sea level as a reference point & assign integers to the points in the set.

UNFAMILIAR ELEMENTS

Challenge 4 Choose the lowest point as the reference point and reassign integers.

Challenge 5 Choose the highest point as the reference point and reassign integers.

Challenge 6 Make a table that compares integers assigned to some of the points in the set for various references.

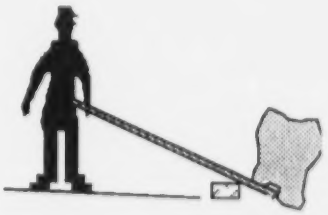
(This type of problem is similar to a common industrial problem, floating reference point.)

EXAMPLE 2: Problems for Problem Solving,
Found In Critical Thinking Approach to Intermediate Algebra

2. Balancing on a teeter-totter is an example of inverse variation; the weight of the two objects and their distances from the fulcrum are inversely related: $W_1/W_2 = D_2/D_1$

Challenge 1 A boy is trying to balance a cement block on a teeter-totter. If the boy weighs 120 pounds, the block weighs 240 pounds, and the block is 6 feet from the fulcrum; where must the boy sit to balance the block?

Challenge 2 A man is trying to pry up a 440 pound stone as shown in the diagram. A block is placed 1 foot from the end of the bar as a fulcrum. If the man is able to exert a force of 120 pounds, how long must the bar be in order to raise the stone?



Challenge 3 If the stone in Challenge 2 weighed 4400 lbs (all other things the same), how long must the bar be in order to raise the stone? Could he do it?

ANSWERS FOR THE ABOVE FOUND IN THE APPROPRIATE INSTRUCTOR'S MANUAL.

EXAMPLE 3: Problems for Problem Solving, Critical Thinking Approach to Beginning Algebra

You receive a base salary of $20,000 per year. If your employer offers you the option of a raise of $1000 at the end of one year and $1000 per year after that, or $300 at the end of six months and $300 each six months after that, which should you choose?

Challenge 1	Guess which option will give you more money.
Challenge 2	Construct a 3-year table showing the 6-month amount of each option.
Challenge 3	Make a second table showing the yearly salaries of the two options.
Challenge 4	Determine any trends in the second table between the options for the first 3 years.
Challenge 5	Estimate the 5th-year amount for each option. 6th-year. 7th-year.
Challenge 6	Write a general equation for the *n*th-year amount for each option.

ANSWERS FOR SOME OF EXAMPLE #3

Challenge 1 Guess which option will give you more money.

Ans. Most students guess the $300 option, because they have been conditioned ...

Challenge 2 Construct a 3-year table showing the 6-month amount of each option.

Ans.

6-month salary	$1000 option	$300 option	6-month salary	$1000 option	$300 option
0 mo.	00,000	00,000	24 mo.	10,500	10,900
6 mo.	10,000	10,000	30 mo.	11,000	11,200
12 mo.	10,000	10,300	36 mo.	11,000	11,500
18 mo.	10,500	10,600	...	...	...

Challenge 3 Make a second table showing the yearly salaries of the two options.

Ans.

yearly salary	1st - yr.	2nd - yr.	3rd - yr.
$1000 option	20,000	21,000	22,000
$300 option	20,300	21,500	22,700

PROBLEMS TO REENFORCE USE OF CALCULATORS OR COMPUTERS

To deny the existence of the use of CALCULATORS or COMPUTERS in the teaching of mathematics in the 1990's would be ridiculous and it would be ridiculous to depend on a calculator or computer to perform all calculations. Developmental math students are often so impressed with the "magic" of a calculator or computer that they feel by using a calculator or computer they can solve any math problem. These students must realize that while calculators or computers are tremendous tools they cannot solve problems without the creative thinking of a student -- involving numbers, operations, formulas, etc. The trick is to incorporate the use of calculators and computers to reenforce the learning and understanding of math; calculators or computers should not be used solely to compute answers to drill exercises.

The use of calculators or computers: allow problems to have more complicated data, require more emphasis on estimation to determine the plausibleness of answers, and allow the investigation of some problems requiring a great amount of computation.

In the texts, CRITICAL THINKING APPROACH TO MATHEMATICS the use of calculators or computers is left to the discretion of the instructor. There are special problems in the *Problems for Problem Solving* which are labeled for suggested use with calculators or computers.

Example 1 of a Problem to Reenforce the Use of a Calculator or Computer

1. Calculate the problem $2 \div 3$ to five decimal places. Can you tell by the answer whether your hand calculator rounded or chopped the result?

   Ans. Yes, rounded 0.66667, chopped 0.66666

2. Calculate the problem $(1 \div 57)$ to five decimal places. Then multiply this result by 100. Can you tell by the answer whether your calculator stores in memory the extra digits of the result it cannot display?

   Ans. Yes, stored rounded 1.75438597, or stored chopped 1.75438596; lost rounded 1.75439 or chopped 1.75438

3. If $1 \div 3 = 0.3333333333...$; and $2\,(1 \div 3) = 0.6666666666...$, then does $3\,(1 \div 3) = 0.9999999999...$ or 1 ?

   Ans. $0.9999999999... = 1$ (both are one)

Example 2 of a Problem to Reenforce the Use of a Calculator or Computer

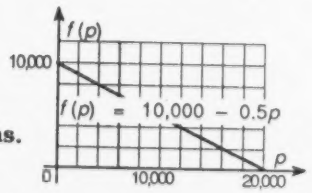
6. In industry a linear function is often used to model quantity/price relationships. The following function represents quantity as a function of price:

   $$q = f(p) = 10{,}000 - 0.5p$$

   where q is the quantity and p is the price per item.

   Challenge 1 Graph $f(p)$.

   Ans.



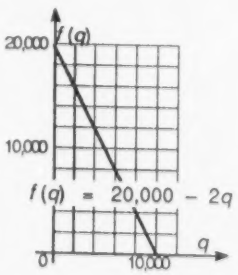
   Challenge 2 Sometimes it is also desirable to know the function that represents price as a function of quantity. These two functions are inverse functions. Determine $f(q)$.

   Ans. $p = f(q) = 20{,}000 - 2q$

   (In industry it is not always necessary to plot the domain on the x-axis; therefore, the variables are not always switched in an inverse function.)

   Challenge 3 Graph $f(q)$.

   Ans.



Example 3 of a Problem to Reenforce the Use of a Calculator or Computer

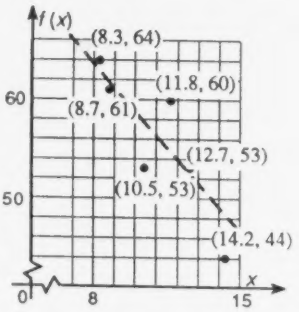
3. Many sets of data cannot be related to a function rule; in such cases we attempt to determine a function that is the "best fit." In business and industry often the best fit for these sets of data is a linear function. Clearly the best fit cannot be used to precisely predict outcomes; but it is very useful in decision making where no other data exists. Suppose that your company builds cars; the sales of a certain model has been declining the last few years. Some officers in the company think that the decline in sales is related to price increases. The following data has been collected:

(8.3, 64), (8.7, 61), (10.5, 53), (11.8, 60), (12.7, 53), and (14.2, 44)

where the first coordinate represents the price that year in thousands of dollars and the second coordinate represents the number of sales of the model in ten-thousands.

Challenge 1 Graph the points of the data set. The graph of such data is usually called a **scatter diagram**.

Ans. It appears that several of the points are nearly collinear, the best fit would probably be a line near the one through (8.3, 64) and (14.2, 44).



Challenge 2 To determine m and b of the linear best fit function, we use a formula from statistics called the **least squares method**. Complete the table at the right.

Ans. Answers appear in the table below in bold type.

Challenge 3 The symbol Σ represents the sum of a set of numbers, hence Σx is the total of the x items. The formulas for m and b

$$m = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2}$$

$$= \frac{6(3624.3) - (66.2)(335)}{6(757) - (66.2)^2} = \mathbf{-2.7}$$

	x	y	xy	x^2	y^2
	8.3	64	531.2	**68.9**	4096
	8.7	61	**530.7**	75.7	**3721**
	10.5	**53**	**556.5**	**110.3**	**2809**
	11.8	60	**708.0**	**139.2**	**3600**
	12.7	**53**	**673.1**	**161.3**	**2809**
	14.2	**44**	**624.8**	**201.6**	**1936**
Σ	**66.2**	**335**	**3624.3**	**757.0**	**18971**

$$b = \frac{\Sigma y - m(\Sigma x)}{n} = \frac{335 - (-2.7)(66.2)}{6} = \mathbf{85.6}$$

Challenge 4 Write the rule for the best fit linear function of this data set, then graph the function. **Ans.** $f(x) = -2.7x + 85.6$; dashed line in graph above.

Challenge 5 There is another statistical formula, **correlation coefficient**, that enables us to determine how close predictions of the data set can be made by the best fit function. The correlation coefficient is represented by r and has a range of -1 to 1. Compute r for this data set if its formula is

$$r = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{\sqrt{n(\Sigma x^2) - (\Sigma x)^2}\ \sqrt{n(\Sigma y^2) - (\Sigma y)^2}} = \frac{-431.2}{\sqrt{159.6}\ \sqrt{1601}} = \mathbf{-0.85}$$

Challenge 6 If r is close to either -1 or 1, then the best fit function is a very good predictor; if r is close to 0, then the best fit function is a poor predictor. How would you rate the calculated r for this data set?

Ans. Reasonably good predictor.

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Section 7 Critical Thinking Instruction and Evaluation

Critical Thinking Instruction

LECTURING (is not equivalent to) TEACHING;
TEACHING (is not equivalent to) LEARNING;
QUIET ATTENTIVENESS (is not equivalent to) LISTENING;
LISTENING (is not equivalent to) UNDERSTANDING.

In the Critical Thinking Approach the **lecture** is a necessary component; albeit shortened to 30 minutes to better fit the attention spans of students. Instructors, based on their experience, should limit their lecture to those topics in a lesson that are the most difficult. CSBM concluded that it is not possible to lecture fast enough to keep a student's brain focused on the topic at hand. Students become passive in a total lecture situation. Shortening the lecture to 30 minutes or less is done to provide the necessary time for teaching, reading, critical thinking, and problem solving.

Another important reason to shorten the lecture is to force students to put more reliance on, and to have more confidence in learning math by reading a text and discussing concepts with other students. The second half of the class period should be a workshop for Critical Thinking Exercises. The advantage of doing the Critical Thinking exercises is the increased student involvement and attention; students become active learners by being involved in open discussions. "Two - way" communication happens through discussions; students should be encouraged and invited to take part in discussions. In the CSBM research we found that far more learning and understanding resulted when students were involved in give and take discussions with peers as opposed to "passive attention" to lectures by instructors.

"When students do not actively think their way to conclusions, when they do not discuss their thinking with other students or their instructor, when they do not entertain a variety of points of view, ..., actively question the meaning and implications of what they learn, ..., they do not achieve higher-order learning. ... . As a result, their adaptability, their capacity to learn on the job and in their personal and civic lives, is severely limited."

Richard Paul, ERIC, 1992

Critical Thinking Math lessons

Critical Thinking lessons can cover more than one day; this is caused by the time needed for groups to complete critical thinking exercises or solutions to problems.

TYPICAL ONE-DAY CRITICAL THINKING LESSON

Day A Students are assigned the reading and vocabulary. (Usually this is a test day of the previous Chapter.)

Day B Students turn in Vocabulary assignment and ask questions about reading assignment. Based on the student questions. Instructor discusses the topics and possible answers. Instructor fills out 1st half of class with lecture on the more difficult topics. The 2nd half is a workshop for critical thinking exercises or problem solving. Students are assigned Drill & Practice Exercises and are assigned the next reading and vocabulary.

NEXT LESSON REPEATS Day B

TWO-DAY CRITICAL THINKING LESSON

Day A Students are assigned the reading and vocabulary. (Usually this is a test day of the previous Chapter.)

Day B Students turn in Vocabulary assignment and ask questions about reading assignment. Based on the student questions. Instructor discusses the topics and possible answers. Instructor fills out 1st half of class with lecture on the more difficult topics. The 2nd half is a workshop for critical thinking exercises or problem solving, but students unable to complete the workshop activities.

Day C Students complete workshop activities from previous day. Students are assigned Drill & Practice Exercises and are assigned the next reading and vocabulary.

NEXT LESSON REPEATS Day B of one-day lesson pattern.

"Mathematics courses are considered to be totally different from other college courses. Passing college courses requires reading and understanding the material presented. However, to pass a mathematics course, an extra step is required: students must prove to the instructor that they can apply what was learned. In mathematics you must understand the material, comprehend the material, and apply the material. Applying mathematics is the hardest task."

Paul D. Nolting, Winning at Math, 1991

TEACHING CRITICAL THINKING

The instructor's role in teaching critical thinking or problem solving is to encourage students to ask questions; it is after all **questions**, not answers, that are the driving force in thinking. The instructor becomes a **responder** to student questions and an **instigator** to generate more student questions and more student interaction. Only when answers generate further questions does thought remain active and alive. The instructor will need to play an active roll in the dicussions and explanations at the beginning of the semester, asking questions that cause students to justify their answers or to think about alternatives.

For Example: Was there something in the way the problem was stated?

What (exactly) are you doing?	Can you describe it precisely?
Why are you doing this?	How does it fit into the solution?
What made you try that?	How will this help you find a solution?

The ultimate goal is to have the students do this. Instructors should not, under any circumstances, provide unnecessary information - spoon-feed -- students.

TEACHING IN THE MATH LAB

Never accept statements, such as, "I can't do this". OR "I don't know what to do".

EXPECT STUDENTS TO HAVE COMPLETED SOME PRIOR THINKING.

Ask questions: What do you understand about the problem?

What information do you need to finish?

However, do not allow students to become frustrated. When new information is needed use: Did you consider ... ? Had you thought about ... ?

The goal in teaching critical thinking is to improve students' reasoning abilities. To what extent do the students engage in various modes of exploring, comparing, analyzing, evaluating, synthesizing, developing, and applying the logic of mathematics; for example, evaluating premisses, analyzing actions, comparing analogous situations, exploring similarities and differences, exploring and evaluating solutions, or analyzing or evaluating proofs. These activities are the indicators of true critical thinkers.

BECAUSE WE ARE SO INTERESTED IN STUDENT SUCCESSS, WE GIVE INFORMATION (SPOON-FEED) ALMOST WITHOUT REALIZING WE DO THIS. THE BEST DISCUSSIONS AND EXPLANATIONS HAVE ZERO INSTRUCTOR INPUT.

Active Learning – Students Working in Groups

It is possible to create positive learning attitudes and help students to gain confidence through a group investigative approach to learning often referred to as **active learning**. We must teach so that students learn to think critically while reading course materials, writing about course materials, taking part in discussions, explaining concepts to other students, and, in general, exchanging ideas with other students and their instructors. We must use the time saved from lectures to present critical thinking exercises. For the second half of a class period, instructors must "shift gears" from the lecture to critical thinking workshops thus increasing the attention span of students; this reduction will also put more emphasis on the responsibility of students to read their text material.

For example, a group of students can volunteer answers to Discussion or Explanation Exercises. Instructors should call on students who seldom or never volunteer. Make sure that students supply reasons with their answers. After the first group completes their answers, call on a second group to analyze, summarize, or disagree with the first group's answers. This will stimulate more active listeners.

It is suggested that class problem solving also be done in groups of 2 - 4 students and the instructor act as a moderator. It is not necessary for the instructor to prevent student groups from "going down the wrong route"; a great deal can be learned from failed attempts. In the Critical Thinking Approach the problem is completed by the group or class in a series of challenges where students are encouraged to verbalize and discuss their thinking processes.

LEARNING IS MORE EFFECTIVE WHEN THE STUDENT ACTIVELY PARTICIPATES IN THE LEARNING EXPERIENCE

C. C. Cole Jr., *Improving Instruction: Issues and Alternatives for Higher Education,* ERIC, Higher Ed. Report #4, 1982

" Among the most frequent of words in the typical classroom ... YOU CAN'T DO THAT. Such is the response long characterizing encounters by mathematics teachers with persistent errors on the part of the struggling student. Actually, these mathematical shortcomings need not end in frustration and defeat. To the contrary, some of the most significant of classroom opportunities stem from careful consideration of student errors. ..."

Francis, Richard L., *The You Can't Do That,* The AMATYC Review, Fall 1994

CRITICAL THINKING SKILLS CAN BE LEARNED MORE EFFECTIVELY WHEN STUDENTS WORK IN GROUPS OF 2 - 4. STUDENTS SHOULD BE ASSIGNED TO GROUPS. SHOULD A DISCUSSION OR EXPLANATION GO ASTRAY, USE DISCRETION BEFORE TERMINATING AND REDIRECTING. REMEMBER MOST OF THE NEW INFORMATION IN MATHEMATICS AND SCIENCE COMES FROM THE POSITIVE EVALUATION AND REHASH OF FAILED ATTEMPTS. MUCH CAN BE LEARNED FROM FAILURE.

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"The cooperative learning section (where students worked together in groups) performed better on the final exam and received higher final grades than the (traditional) lecture section. ... The most pronounced effect was the contrast between the number of students who successfully completed the course. (2 students failed the cooperative section and 12 in the traditional)."

Keeler, Carolyn & Mary Voxman, *The Effect of Cooperative Learning in Remedial Freshman Level Mathematics*, The AMATYC Review, Fall 1994

NEED FOR STAFF TRAINING

As stated earlier in this booklet; probably the most Critical Need when implementing curriculum reform is that of staff training. In order to use the Critical Thinking Approach to Mathematics it is necessary for the staff to understand the changes in both curriculum and pedagogy. The need for training sessions may be particularly essential for those colleges and universities that use adjunct faculty or teaching assistants to cover these courses because these instructors are often so entrenched in their own teaching patterns. There is also a need to provide additional training for lab assistants for these courses.

PEER TEACHING

Repeating an item that appeared earlier in the booklet; when student success is a high priority, upperclass math students should not be overlooked as an efficient resource of quality instruction; their teaching often provides a higher rate of student success. They are a cost effective means of increasing individualized or small group instruction. They have a better understanding of developmental student learning problems and, in general, they are more sympathetic to these problems.

Unless they are trained and supervised, students used as peer teachers should not be assigned to a group larger than 6 to 8 developmental students. Peer teachers should have course materials to follow; they should not be expected to generate materials (other than quizzes) on their own. Peer teachers should be monitored by a qualified instructor or professor and ample opportunity should be provided for peer teachers to meet with monitors to have their questions answered and to provide other support as needed. A word about upperclassmen working as peer teachers or as tutors in a math lab. We found in the CSBM that most of these upperclassmen needed to be taught how to respond; they were far too eager to "tell all - show what they knew" or "they did not really understand why the remedial student had a question and needed help - thus they provided very little". After training most of the upperclassmen did an excellent job, because they were so much closer to the time when they themselves were learning the same concepts, the tutors seemed to know exactly what question to ask.

"Regularly question your students Socratically: probing various dimensions of their thinking: their purpose; their evidence, reasons, data; their claims, interpretations, deductions, and conclusions; ... In general design activities and assignments so that students must think their way through them. ..."

Richard Paul, Foundation for Critical Thinking, 1993

UP THE STREET OF BY AND BY

ONE ARRIVES AT THE HOUSE OF NEVER

Cervantes

HOW IS CRITICAL THINKING EVALUATED?

SINCE STUDENTS STUDY ONLY THOSE TOPICS AND FACTS
ON WHICH THEY ARE TESTED
GRADING MUST INCLUDE EVALUATION OF CRITICAL THINKING.

Homework: Completeness / Correctness, Study Partner.

Class Participation, Subjective evaluations, Credit for:

Group effort or Individual effort put forth in class for participative and discovery exercises;

Innovative, creative, or different suggestions offered in class; using the criteria for evaluation of mathematical reasoning -- clear, precise, specific, accurate, consistent, logical, and complete.

Class Tests: Quizzes, Chapter Tests, Criteria Reference Tests (CRT),

Attendance: 10% of grade in developmental math courses.

IF WE EXPECT STUDENTS TO LEARN TO THINK CRITICALLY, THEN WE MUST REWARD CRITICAL THINKING EFFORT WITH A GRADE.

UNFORTUNATELY MANY TEACHERS AND INSTRUCTORS HAVE A DIFFICULT TIME EVALUATING DISCUSSIONS AND EXPLANATIONS.

Because the majority of teachers and instructors have had little or no training in techniques of evaluating discussions or explanations they have not developed skills in this area as fully as evaluation techniques developed for other types of exercises. The criteria for evaluation of mathematical reasoning are, to what extent is the reasoning:

clear, precise, specific, accurate, consistent, logical, and complete.

Not all of these standards will necessarily apply to each situation and students should be aware that these standards are being used to evaluate their reasoning.

Evaluations need to include instructor subjective opinions. We found in CSBM that an instructor's subjective opinion of student success in future courses was a far better predictor than test scores or grades. For many years we have been bombarded with information claiming that subjective grading was unfair; this simply does not have to be the case. Effort made on participative and discovery exercises should be included as part of the grade. If critical thinking is to be evaluated, then a substantial part of the grade must come from contributions of the student to the class effort in learning; contributions to discussions, explanations, and discovery exercises, as well as questions asked in class and answers to questions given by the student in class.

A set of 3 by 5 index cards can be used to select students for Discussion or Explanation Exercises and grading comments can be written on each student's card. A scale from 1 to 5 can be used to indicate quality of response; keep in mind the Critical Thinking Standards (clear, precise, specific, accurate, consistent, logical, and complete) in ranking students.

In the CSBM we encouraged instructors to put considerably less emphasis on homework (suggesting that homeworks are learning exercises and could (should) be done by groups of students) and instead to use short one-topic quizzes for class evaluation. The reason for this was to reduce the amount of grading time spent on homework and to gain, through the use of quizzes, a better feeling on student understanding of concepts presented for the instructor. In addition we used department chapter tests and a department final. Student progress was based on mastery of the chapter tests, and a combination of all tests and other evaluations were used for a letter grade.

TRADITIONAL TESTS

A. H. Schoenfeld calls traditional teaching and testing with traditional tests SCHEMATA FITTING. Schoenfeld states that in his view the primary accomplishment of mathematics education in the United States today is the teaching of schemata fitting, not understanding mathematical concepts. The use of traditional tests based on the memorization of algorithms does not evaluate the understanding of mathematics. Additionally, the typical word problem is not, in most cases, a problem that allows problem solving skills to be developed; instead traditional word problems tend to foster schemata fitting.

Dr. A. H. Schoenfeld, University of California, Berkely, *Some Thoughts on Problem-Solving*, Research and Mathematics Education, 1982

To bring the problem of schemata fitting versus concept understanding to a more understandable level, Schoenfeld has proposed simple tests for various grade levels.

$$\frac{137 + 137 + 137 + 137 + 137}{5} =$$

The test above is for high school or for college students. For best results, prepare student copies in advance, fold in half and administer as a timed quiz having students show all work and note the time the student needed on the quiz. As you can guess, what you are really interested in is not the time it takes nor a correct answer, but the number of students who first needed to add the numerator to get 685 before they could divide by 5. According to Schoenfeld, the students who do this are schemata fitters because they are taught the schemata which adds numerators first then divide the numerator by the denominator. The critical thinkers, on the other hand, will note that the numerator contains five 137's and realize that repeated addition is multiplication ($5 \cdot 137$) and that the two 5's will cancel leaving 137. You will be surprised at the number of schemata fitters you have in class.

A quiz for elementary students is shown below.

On a ship there are 25 sheep and 2 goats, how old is the captain?

If the student is in grades 1 or 2 then the Captain is 27 years old because you always add two numbers. If the student is in grades 3 to 5 then the Captain is 50 years old because you always multiply two numbers.

Section 8 Topics Related to CRITICAL THINKING

A VARIETY OF INSTRUCTIONAL MODES

Since students have different approaches to learning, it is necessary to have available some flexibility in the presentation of mathematical concepts. The CSBM research revealed that 80 - 90% of the students learned effectively in the lecture / critical thinking workshop instructional mode. However, for a few of the students it was necessary to have alternate methods available.

SELF-PACED MODE

In every program there is a small percent of the students who are highly motivated and are capable of moving faster than the group, or there are some individuals, usually returning older students, who are faced with outside responsibilities that demand a great amount of their time. It is advisable to establish a self-paced mode for these students who study from the texts, attending class or lab as necessary. To ensure success for this component it is necessary that these students demonstrate periodically that they are on or ahead of schedule.

MATH LAB MODE

One of the most effective components of the CSBM program was the math lab where live tutors were available. The CSBM research revealed that the most important component of supplementary instruction in a developmental math course is a Math Lab. In the CSBM, the lab was open Monday afternoon, Tuesday - Thursday 10:00 am. -8:00 pm., and Friday morning. Other components in the Math Lab, such as CAI, AV, etc. offer help for some students. Lab tutors were peer teachers, students in mathematics or secondary math education. A teaching assistant or instructor was assigned to be available to help the upperclass tutors when needed.

ELECTRONIC MODES

At the time of the CSBM research, electronic modes of instruction were not as well developed or as available as they are today. We found that 5% - 10% of the students learned effectively, without supervision, using the electronic instruction that was available. This was not overly encouraging since we found that 75% of these students could learn almost any way we tried to instruct them.

The greatest complaints of the students who attempted, but gave up on electronic instruction, was the time it took to find answers to questions, the time needed to run a program (both of these are related to the problem of the inability for branching in current computer programming), or the time to get a machine to use. Most of these complaints could be resolved by asking or working with a lab tutor. Certainly as these modes are developed they should be made available to students who can. learn effectively using them; however, it is a great mistake to have electronic instruction as the only mode of instruction for students.

Much of the existing software and tapes are either poor quality or very expensive and almost all are linear. As cited earlier, there are many good uses of electronic instruction. Recent developments in the fast moving field of computers, in particular the incorporation of a CD disc, will allow for creation of programs that will change the role of the instructors and professors from that of a lecturer to a class of students to a consultant for a very large number of computer taught students. Within a few years CD computers will become the major source of "information giving", not instructors or professors as in the past. As a result, far fewer instructors and professors will be needed, saving universities and colleges payroll dollars. Surviving instructors will have changed their roles to one of responding to and instigating student questions.

OTHER AREAS THAT EFFECT THE LEARNING OF MATHEMATICS

STUDENT PLACEMENT

Dr. Paul D. Nolting states: "I have conducted research on thousands of students who have either convinced their instructors to place them in higher level math courses or have placed themselves in higher level math courses. ... These students failed their math courses many times before realizing they didn't possess the prerequisite cognitive entry skills needed to pass the course. This is evident by the many students who repeat a higher level math course many times before (as opposed to) repeating the lower level course."

Nobody wants to review a math class even if that is where they belong. However, nothing wastes more time for everyone involved than an incorrectly placed student. The CSBM research revealed that a very high percentage of students fail to pass remedial math because they attempt courses requiring more math background than they possess. Probably the greatest barrier to having a successful remedial math program is the inability to place students in the proper course. Students placed in courses above their capability usually waste more time trying, again and again, to pass that course than they would lose taking a lower level course to strengthen their math background.

On the other hand, it is not beneficial to place students in a lower course when they may have deficiency in only one or two areas. For example, it is not unusual to have a group of students whose placement test results indicate that they are ready to start Beginning Algebra, except they do not have an adequate background in Fractions. To place these students in Arithmetic would not only be unfair, but probably would cause most of them to fail because of boredom. A better course of action would be to place them in Beginning Algebra and provide independent study in the math lab to correct their weakness in fractions.

ATTENDANCE

Another great barrier to having a successful developmental math program is skipping classes by students. With the cost of taking a college course it is difficult to reason why students will not attend a class; some of the students have grants, others feel that such a math course is beneath them and that they can pass the course without much effort.

It is helpful if the college or university provides some dignity to the course by allowing credit. Many schools give credit which counts toward the overall GPA, but does not count toward graduation, or is counted as an elective. In a CSBM survey we were surprised at the types and number of courses for which students in colleges and universities could earn graduation credit; but the same schools denied credit for much more academically difficult developmental math courses.

Students who did not attend class often became a **negative influence**. The last few years the CSBM program existed under the condition that students who did not attend classes could be dropped from the program; this possible course of action had a great positive affect on student attendance. However, the first years the CSBM existed in a situation where students who did not attend could not be dropped from the program. To remove the "negative influences" the first years of the program, students who did not attend were placed in the Self-paced mode of the program. These students were often pleased with this placement, seemingly not realizing that their own inaction in a self paced program lead to their failure.

STUDY HABITS

Most of your students will not be pursuing math or science related degrees and math is very different from other nonscientific courses; math is a course that continually builds on previously learned concepts. It is imperative that students keep up. In this respect, students should attend class and keep up with homework. It is considerably more important that homework be 100% attempted than 100% correct. A study partner is advisable, students who study in pairs (or threes) usually do better in a developmental math class; understanding is more important than a perfect homework paper.

SELF-IMAGE

Most of the students in a math remedial program have a very low opinion of their ability to do mathematics. These students have often suffered embarrassment and humiliation in preveious mathematics situations. It is important that this condition does not continue.

Success of a developmental program is contingent upon the success of the students in the program.

Real student motivation is closely related to a student's success.

"SUCCESS BREEDS SUCCESS"

MATH / TEST ANXIETY

Most schools now have counseling services available to help students who have "Math Anxiety". Math anxiety (particularly in tests) is a real problem for some students and special attention should be provided for these students. In the CSBM program, students with math anxiety were put in help groups, assigned a study partner, and/or were given special help in the lab.

Building test confidence takes time and patience.

There are currently several programs experimenting with reducing test anxiety by conducting special sessions on how to analyze and take a test. Many of these programs use sample tests to help students in: determining what a question asks, identifying wrong choices in a multiple choice test, and defending the choice the student selects as the correct answer. Care must be exercised that the goal of the program is to help students learn how to *take* a test *not teaching* a test.

> "Results of this study suggest that agreement with some math myths can be reduced with more mathematical preparation of students. We need to encourage and motivate students to take more math courses beyond the required minimum. ... my experiences suggest that employing a variety of teaching strategies in classes could be helpful in dealing with math myths, such as math anxiety, ..."
>
> **Odafe, Victor U., *Student's Perceptions of Myths About Mathematics*, The AMATYC Review, Spring 1994**

The Role of Memory in Math Instruction

Memory plays a positive important role in any learning situation; however we have arrived at a point in educational time where too much stress is placed on students to memorize. When students memorize algorithms and pass tests; the assumption is that they therefore know and understand the math concepts contained in these tests; there is ample research data to show beyond question that this is not the case. We occasionally change from one traditional textbook to another all of which are geared to teach the memorization of math and most of us admit that there is really little difference between any two of these texts. What has been universally recognized is that the deterioration in the quality of American schooling is due to an over reliance on memorization, too much trying to cram facts and information into students' heads and not giving them enough time to understand what they are learning. Over stress on rote memory is caused, to a great extent, by the excessive amount of material presented in a math class. Students feel that to pass a math test their only hope is to memorize sufficient information; "short - term - mail - box" information storage.

FLEXIBILITY

Successful Developmental Programs require flexible and creative management. Most colleges and universities are still well entrenched in management policies of the "pre-computer" age. As the current saying states: "We talk the talk, but we do not walk the walk." The use of a computer based management system could change the answers to many of the following questions.

How flexible is the placement of students?

Are all students required to start at the beginning of a course even though they may already know and understand half of that course?

Can students complete partial courses? Can students take a "Half-course"? Can students complete a course early?

Has your department ever offered partial courses? A course of just 1 chapter?

Can your department offer partial course tuition or credit?

Can students take a retest on a failed chapter? How many times?

How soon can students repeat a failed course or chapter?

Must they wait until the following semester?

How many times can students repeat a failed course? Are there limits?

Can students not making an effort or not attending classes be identified? Are there provisions for these students?

What provisions are made to individual questions? Individual instruction? Instructor office hours ample and convenient for students?

Pacing the Critical Thinking Math Course

Check your students' placement exams. to determine if you need to spend time teaching topics of the text. You could use quizzes on the topics for any of the Chapters skipped to be sure the students understand the math. If you determine by a quiz or other means that some topic(s) contained in skipped Chapters are not understood by students, insert the appropriate lesson(s) into your teaching schedule.

Pacing a course is a two-edged sword, don't waste time presenting topics the students already understand; on the other hand, be very calloused to their verbage of how much they already understand because they took this course and passed with very good grades in high school.

SHIFTING TO A MORE *STUDENT INVOLVED* MATH COURSE

To aid the implementation of the Critical Thinking Approach to Mathematics; some of the points below should be discussed with students during the first few days of the term.

What is the main reason that students enroll in college to obtain a degree?
It will allow them to attain a better paying job and use that training to advance to a higher pay.

Business organization can be thought of in three levels: top management, middle management, and the worker; which level interests you the most?
Our principle responsibility is to educate students who will become management.

Why is it that our scientists create the "chip" and originate many other inventions; other countries manufacture these items, gaining most of the profit?
To win this economic war the U.S. must produce what it creates.

In this day of rapid change, which graduates will succeed in coming out ahead?
It will be those who are able to adjust, extend, and create, while doing their jobs; this must be done to help their companies produce better products and it must be done without supervision.

If a graduate is unable to read, think, and act on their own to solve the everyday problems to successfully complete their work (graduate needs supervision), will they keep their job?
No! A company then has to pay two salaries to complete one job.

Moving to a more student involved course supported by Critical Thinking Approach to Mathematics can be thought of as taking part in an experiment that will better prepare your students for employment. Students will learn how to read technical material such as math; be encouraged to build reference resources; and they will be challenged to think and reason. Because of the shift to a student involved learning, depending upon the effort your students are willing put into their learning, they should be able to keep a job more easily after they find one and quite possibly obtain a higher salary.

The student who will be most successful in keeping a position and obtaining promotions is the one who learns how to find, read, and understand reference material, then use the acquired information to solve problems.

IT'S THE HIGH SCHOOL'S FAULT

These words are often stated in discussions concerning remedial courses in math at the college or university level. It certainly is true that if high school students were better prepared remedial courses in math at the college or university level would not exist. However, those of you who think that a simple solution can be effected should volunteer to teach a high school math class for one semester. Suppose that you were to teach a Beginning Algebra class of 30 students; but 22 of these students did not want to learn -- seldom, if ever, did any homework -- paid little, if any, attention to your lecture. Could you fail this percentage of students and keep your job? Actually this theoretical class is quite likely to have 40 students and 35 do not want to learn. I volunteered and learned that in today's high school setting students feel that there are many more important things to think about than mathematics. I have adopted a different attitude toward the math preparation of high school students.

If we at the College and University Level Cannot Successfully Teach Remedial Math to Students, then we have No Right to Condemn High Schools.

Because we are in a better educational situation, we must show the way by creating a method for successfully teaching nonmathematical students developmental math.

ARE THESE STUDENTS INTELLIGENT ENOUGH TO LEARN MATH?

The answer is YES.

IS IT POSSIBLE FOR THESE STUDENTS TO RELEARN HIGH SCHOOL MATH IN COLLEGE OR UNIVERSITY?

Again the answer is YES.

CONCLUSION

Reversing the dependence on rote memory to pass math tests is not an easy task. Nonmathematical students should be learning and understanding mathematics in order that they can reconstruct math concepts as needed after they leave college. Nonmathematical students should be screened and properly placed, time is too precious to waste. The attitudes toward math must be improved. While it is not the intent to make students "love" math, students need to become aware of its importance in their lives and be comfortable working with mathematics.

The CSBM researchers found that reducing the lecture time and increasing the discussion time was the most beneficial method of increasing student success. Decreasing the lecture time required students to do more reading of the text to gain information, which, in turn, led to more questions and discussions. The advantages of students reading texts for information is obvious. To emphasize importance, vocabulary questions were also included in tests. The advantage of discussions over lecture is the student involvement; more students become "involved" in open discussions, while many are "passively attentive" during lectures. Students should be encouraged and invited to take part in discussions. Instructors must listen and discuss all student questions.

There is no "dumb" question.

Based on the research, the Critical Thinking Approach to Teaching Mathematics was developed to provide better tools for teaching the nonmathematical college student. Other areas of math instruction needing improvement were identified (reading and critical thinking) and a total instructional program was developed.

For the student, reasoning is not a matter to be learned once and for all, then forgotten. Reasoning is a matter of lifelong learning, a matter of bringing insightful organization into the patterns of thinking and action.

For the instructor or professor, reasoning is a matter of learning how to design instruction so that students take command of the logic of their own thinking and learn to analyze their thinking processes for improvement.

Your "I Will" is more important than your "IQ"

THERE IS NO PANACEA.

As Instructors, Professors, and Administrators, we must lead and not follow others in initiating and making changes in education. We must not allow educational formats to be dictated by outsiders. The easiest thing to do is to continue doing what we have been doing (business as usual); let inertia and lack of momentum take over. Unfortunately, this course could cause drastic change in the role of colleges and universities as well. Lack of change will force outsiders to redefine the roles (if any) that instructors and professors will play for the future in education.

BEST COPY AVAILABLE

YOU SHOULD BE AWARE OF THE FOLLOWING

According to some of our peers (Instructors, Professors, and Administrators), people in Business , and some people in government "The greatest obstacle to changing Developmental Mathematics is caused by the person in charge of the classroom being afraid of change."

We must not be so afraid of change that we are willing to live with failure. We in education, particularly in developmental mathematics courses, are part of the new world economy of change: the mathematics background of students entering college has changed drastically, their attitudes toward learning have changed, and their reasons for coming to college have changed. We cannot turn time back; we must become part of the changing world, we must change instruction and materials to meet existing conditions. We must lead our students into their "technical world" of change.

Surveys indicate that 95 - 98% of the Educators, in particular Math Educators, respond that they would not make any changes in curriculum or in their instruction methods even when they feel the current curriculum or method of instruction is not working and their rhetoric most often supports the need for change.

WHY? Why are we so afraid of Change?

From the "Scans Report for America 2000"

Schools for Tomorrow Should:

Focus on development of thinking and problem solving skills as well as basic skills,

Students learn cooperatively as well as individually, and

Incorporate lecture and active learning to allow students to learn to think.

I began with a Section that contained comments from CEO's of large companys. I would like to close with one by **Mr. Bob Allen CEO of AT&T June, 1996** : "When you are moving in the fast lane and the forces of fundamental technological change come roaring up behind you, you have two choices. You can pull over to the side of the road and let history pass you by. Or you can step on the gas and make history yourself."

At first I said great thought, then as I reflected further I became aware of a real possible danger. You have all experienced that frustrated feeling when roaring down an interstate in an area that is unfamiliar to you and you misread an exit sign. Miles later you are trying to figure out how to get back on the correct highway and make up that lost time.

I am very concerned in that, without exception, successful experimenting and research in education with technology is focused on our top 10-15% who can learn anything, any way they are taught. In our rush down this super highway of technological change let's not accelerate right past the correct educational exit; for if we do we might end up with 10-15% of super educated and all of the rest of the population illiterate. We must teach everyone the 4R's (*Reading, Riting, Rithmetic, and Reasoning*) so they too can participate in the new economy. Failure could lead to a potentially extremely dangerous political situation.

The easiest thing to do is to continue doing what we have been doing (business as usual); let inertia and lack of momentum take over. Unfortunately, this course could cause drastic change in the role of colleges and universities.

We must overcome accelerated instruction forcing students to memorize algorithms. We live in a time of change, we must endorse and support change that will improve education in developmental math.

CSBM Critical Thinking
The ability and willingness to ask questions and to seek answers toward the positive solution of a problem or situation which demands thinking that is self-directed, and self-corrective.

It looks like we are faced
with an
insurmountable challenge

Pogo

VITA
Dr Melvin Poage

TEACHING EXPERIENCE

Seven years inner-city Junior High School.
Six years suburban High School.
One year Mathematics Coordinator K - 12 system.
Fifteen years University Professor.

OTHER WORK EXPERIENCE

Three years Senior Mathematics Editor for SRA Publishing Company.
Four years Executive Mathematics Editor for Addison Wesley Publishing Company.
Three years Editor-in-Chief for Addison Wesley Publishing Company International.
Nine years President of Materials & Technology for Education Limited.
Mathematical Consultant: (including) New York Pub. Sch.; East Lansing Pub. Sch.; British Columbia Schools; Texas A & M Univ.; University of Texas, San Antonio; Haskell Indian Nations University; University of Denver; East Texas State Univ.; University of Kansas, Salina; University of Missouri, Kansas City; Newbury College; Angelina College; Addison Wesley Pub. Co. of Canada; Addison Wesley Pub. Co. of U.K.; Prindle, Weber & Schmidt Pub. Co.

ACADEMIC CREDENTIALS

Bachelor of Science in Mathematics Education; University of Denver.
Masters of Arts in Educational Administration; University of Denver.
Masters of Science in Mathematics; University of Colorado.
PhD in Mathematics: Michigan State University.

PUBLICATIONS

Textbooks published with: Addison Wesley; Prindle, Weber & Schmidt; Kendall Hunt; and Materials & Technology for Education.
Articles in: Arithmetic Teacher; Mathematics Teacher; Ontario Teacher of Mathematics; Research in Mathematics; The Critical Thinking Journal; and ERIC.

PROPOSED EDUCATIONAL OBJECTIVE FOR THE 21st CENTURY (the AGE OF CHANGE)

TO MAXIMIZE THE FUTURE POTENTIAL OF OUR STUDENTS
OUR GOAL MUST BE TO EDUCATE STUDENTS WHO CAN DETERMINE
WHAT IS NEEDED, LEARN TO FIND REFERENCE MATERIAL,
READ AND UNDERSTAND THE REFERENCE MATERIAL,
EVALUATE THE ACCUMULATED DATA,
THEN USE THE ACQUIRED INFORMATION TO SOLVE PROBLEMS
(NOT JUST MEMORIZE ALGORITHMS).

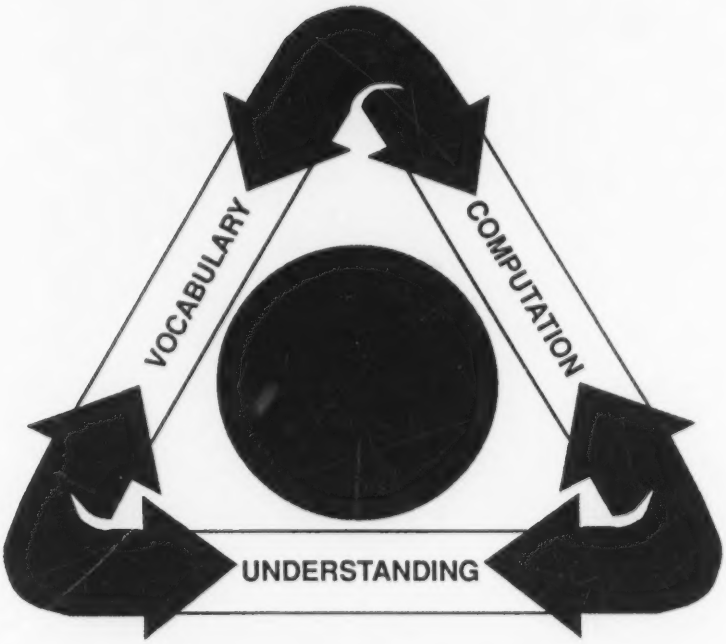
M.T.E. Ltd.

Materials and Technology for Education

3095 South Trenton Street

DENVER, CO 80231 - 4164

(303) 696 0839



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